Robust Design and Optimization of Thick Film Accelerometers in COMSOL Multiphysics with OptiY

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Abstract: Optimization of the design parameters with regard to the tolerances is an important purpose of the design process. We used a COMSOL Multiphysics structural mechanics model with the OptiY tool for finding an optimized design of a thick film accelerometer conforming to our sensitivity and cross sensitivity requirements, inclusive of resonance frequency. We calculated the probability of a system failure due to relevant tolerances of the design parameters previously found out in a sensitivity analysis. In the final step, we will be able to calculate a robust design of the ceramic thick film accelerometer. As a result we will obtain a design optimized for a set of functional requirements and design tolerances.

Keywords: Tolerance Analysis, Robust Design, Probabilistic Design, Optimization

1. Introduction

In virtual design and development of technical products, all design parameters must be specified so that requirements of manufacture, customer, and services are met. A serious problem are the variability and the uncertainty of design parameters, called tolerance, caused by manufacturing inaccuracy, process uncertainty, environmental influences, abrasion, human factors, etc. Classical simulation cannot predict all the variations of the system behavior that are caused by tolerances. A tolerance analysis calculates the probability distributions of functional variables from any type of the probability distributions of the design parameters. This enables the reliability of the system to be deduced. Generally, the ideal design found by way of nominal optimization is not insensitive to tolerances of the optimized design parameters. In order to find a robust design, i. e. a design the functional behavior of which is only little affected by tolerances, optimization for robustness has to be performed.

These methods are provided by the analysis and optimization tool OptiY [1]. We have used these methods for optimizing the design of a new accelerometer made of low temperature cofired ceramics (LTCC).

2. Tolerance Analysis for Robust Design

Any design parameter can be modeled as a nominal value and a probabilistic distribution in a tolerance range. Most physical variables and design parameters may thus be viewed as random variables and have to be controlled to achieve reliable products [2][3]. Classical deterministic simulations deal only with the mean or nominal values of the design parameters, whereas a tolerance analysis or a probabilistic design study takes into account also their probability distributions.

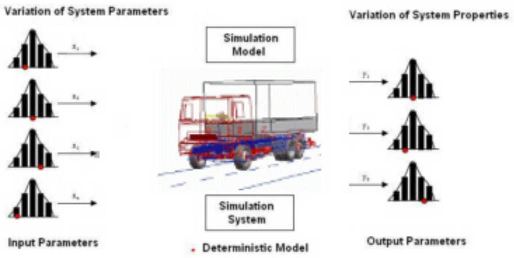

Figure 1. Principle of a tolerance analysis.

This is shown in figure 1. The probabilistic distributions of the system properties are deduced from the distributions of the system parameters through a deterministic system model. This makes it possible to design the system for needs of reliability in conformity to the specifications and hence to maximize safety and quality, and to minimize rejections and cost.

2.1 Numerical Basics

State of the art for all software systems available on the market is the Monte-Carlo simulation [6]. In this method, for every input parameter a sample size is generated. With each of the samples, a deterministic simulation is carried out to get output variables. Finally, a statistical evaluation of these calculations provides the desired probabilistic distributions of the output parameters. Unfortunately, the Monte-Carlo method is computation-intensive when a representative sample size has to be calculated.

As an example, figure 2 shows different kinds of Monte-Carlo simulations with four random parameters. The standard deviation of the output probability distribution converges for a sample size of approximately 1000. However, the sample size required for acceptable results increases exponentially with the number of random variables. In practice, the available computing power for Monte-Carlo simulations is often insufficient.

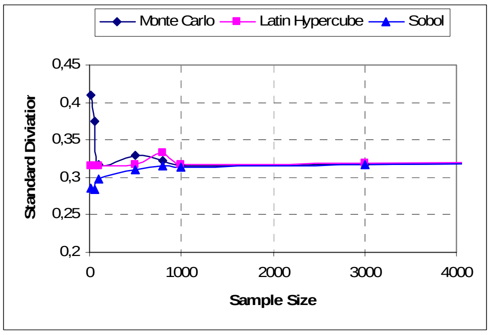

Figure 2. Monte-Carlo Simulation [1]

A way out is the use of analytical methods that are faster compared to Monte-Carlo simulation. One of those methods is the Second-Order-Analysis based on the second order Taylor series:

$$f = f_0 + \sum_{j=1}^{n} \frac{\partial f}{\partial x_j}(x_j - x_0) + \frac{1}{2}\sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_j^2}(x_j - x_0)^2$$

It is an analytical calculation of the probabilistic distribution of the output variables, i.e. their center moments (mean, variance, kurtosis and skewness) are deduced from the center moments of the input variables. Based on these center moments the probability distributions of the output variables are approximated. For calculating the Taylor series coefficients, the number of calculations is $\mathbf{2n^2+1}$, where n is the number of random input variables. For four random variables, only 33 model runs are required, compared to about 1000 calculations of a Monte-Carlo simulation.

2.2 Sensitivity Analysis

With a sensitivity analysis, the system complexity can be reduced and the cause-and-effect chain can be explained. In particular, it can be found:

- The contribution of each design parameter to the function variability,
- Insignificant parameters for eliminating them from the final model,
- Interaction between the parameters.

The partial derivatives averaged over the tolerance interval are sometimes regarded as local sensitivities. As they are calculated only at the upper and lower boundaries of the tolerance interval, their significance to the influence of a model variable is pretty small. Generally, the influence of a design parameter is not constant over the tolerance interval. Therefore, a global variance-based sensitivity method with Sobol´s index has to be considered. The main and the total effect are calculated. The latter also includes the interactions between the tolerances of the input variables, which are calculated by pairwise combination in OptiY.

2.3 Reliability Analysis

Often the variability of the design parameters results in an inoperable system. The constraint boundary violations of the output variables due to tolerances are investigated in a reliability analysis (Figure 3). The reliability requirement is met, if all of the functional properties are inside the acceptable ranges even if the design parameters scatter. The ratio of inoperable solutions to all scattering solutions is called failure probability. For a design found by a nominal optimization a failure probability of about 50% is expected if the optimum is located on a boundary. Such a design has to be changed such that a lower failure probability is achieved, at best about zero. This is performed by a robustness analysis.

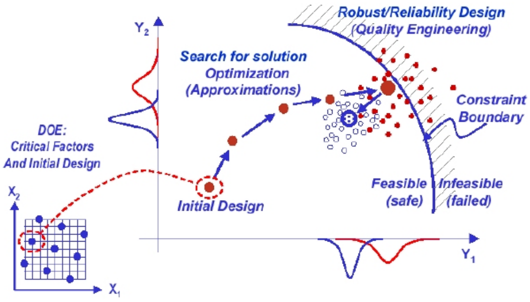

Figure 3. Reliability Analysis [6]

2.4 Robustness Evaluation

To find a so-called robust design with a low failure probability which is only little affected by tolerances, is the aim of a robustness evaluation (Figure 4).

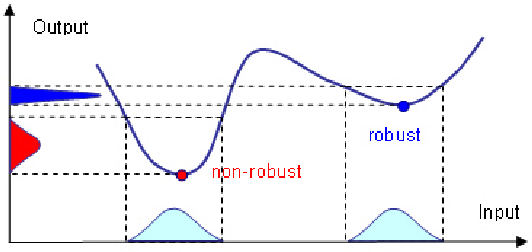

Figure 4. Robustness Evaluation

For this purpose the influences of the tolerance of each design variable to each functional variable has to be estimated. It is advantageous if only the main effects found in the sensitivity analysis have to be considered. This allows to apply the reduced second order analysis for computing the output variances and to reduce the computational effort. The design solution is robust if the output variance is small, that is consistent quality of the product under all conditions is ensured.

3. Robust Design of a Thick Film Accelerometer

Today's accelerometers made in thin-film technology offer sufficient functionality in a cost-effective way. Nevertheless, thick-film accelerometers made of Low Temperature Cofired Ceramics (LTCC) are of interest, since they promise a higher temperature range and lower costs in small-series production. The working principle is based on a seismic mass *M* arranged on two parallel leaf springs *S* which carry piezoresistors *P* connected to form a measuring bridge (Figure 5). An acceleration in the z-direction to be measured is transformed into a change of the bridge voltage U_b.

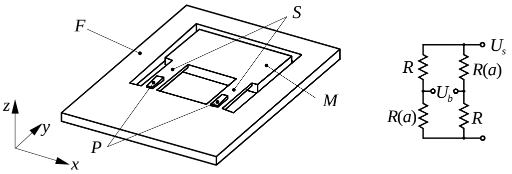

Figure 5. Basic Design of the Thick Film Accelerometer

The LTCC technology and all its specific problems of structuring, printing, stacking, laminating, and firing will not be considerd in this article. We focus on the best design of the accelerometer with regard to the dimensional accuracy of LTCC.

3.1 COMSOL Multiphysics Model

A COMSOL Multiphysics structural mechanics script model contains all mentioned elements, with the exceptoin of the electrical connections of the circuit of the bridge (Figure 6). All material properties are constant. In practice, the accelerometer is bonded to the lower surface of the frame. In the model this boundary is set fixed while all other boundaries were free.

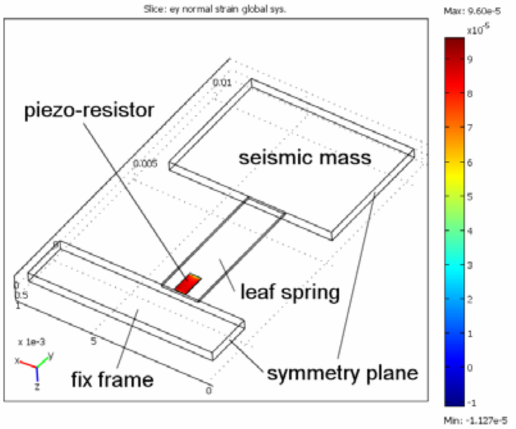

Figure 6. COMSOL Multiphysics Model of the LTCC-Accelerometer

For simplification we assume mirror symmetry of the geometry inclusive of the tolerances. Thus only one half of the sensor is modeled. Furthermore the accelerometer should work far from resonance, therefore the electrical output can be calculated from a static model. The mean normal strain in y-direction in the piezo-resistors e_{ym} is a measure for the change of the resistance of the piezo-resistors ΔR multiplied by a constant factor *k*. Therefore we obtain for the sensitivity of the accelerometer *S* under an acceleration in z-direction a_z depending on the bridge voltage U_b and the feeding voltage U_s:

$$S = \frac{U_b}{U_s \cdot a_z} = \frac{\Delta R}{2R \cdot a_z} = \frac{e_{ym} \cdot k}{2a_z} .$$

Equally we get the cross sensitivity CS for accelerations in the x- and the y-directions, where the acceleration in the direction of y is more critical:

$$CS = \frac{U_b}{U_s \cdot a_y} = \frac{\Delta R}{2R \cdot a_y} = \frac{e_{ym} \cdot k}{2a_y}.$$

The model calculates both S and CS, and the first resonance frequency f_R as a further essential property. It has about 40.000 DOF's.

3.2 Nominal Optimization

In the first step, we used the COMSOL Multiphysics model with the OptiY tool for finding an optimized design conforming to our sensitivity and cross sensitivity requirements, inclusive of resonance frequency. For these purposes the following design parameters are set as input variables for the optimization process (Figure 7):

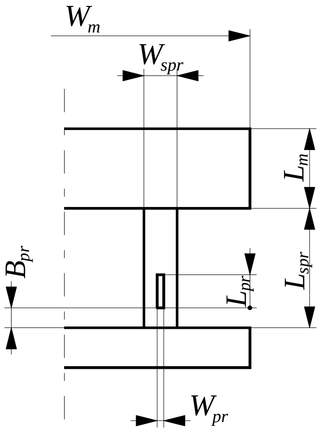

Figure 7. Input variables for the nominal optimization of the LTCC-Accelerometer

- length L_{spr} and width W_{spr} of the leaf springs,
- length L_m and width W_m of the mass,
- length L_{pr} of the piezo-resistor and distance B_{pr} between it and the frame.

After about 350 runs of the FE-model inside of the Hooke-Jeeves-algorithm the optimization converges. Figure 8 shows the development of C and f_R over the number of iterations.

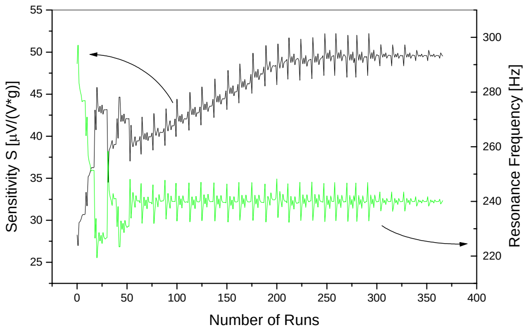

Figure 8. COMSOL Multiphysics Model of the LTCC-Accelerometer

As a result we get the set of design parameters that fulfill the restrictions and functional demands at optimum.

3.3 System Failure Analysis

As an example, the tolerances of the design parameters W_{spr}, L_{spr}, and B_{pr} have to be defined for the sensitivity analysis. We assume normal distributions, but any kind of distribution may be involved. The sensitivity analysis calculates the distribution densities of the functional parameters C, CS, and f_R (Figure 9) by the second order analysis method. A red area stands for a behavior outside the acceptable range.

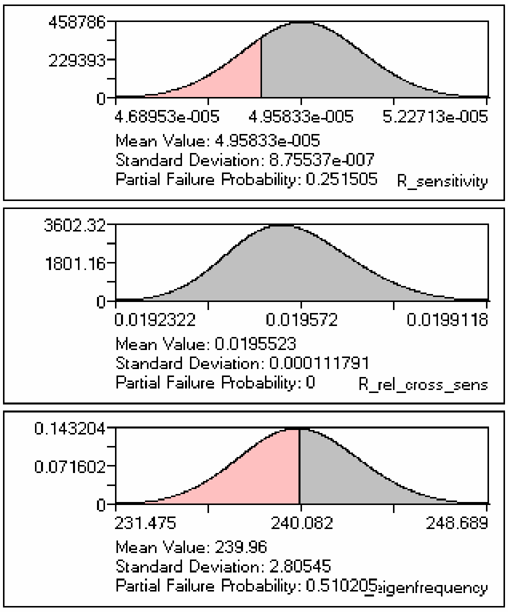

Figure 9. Distribution Densities of S, CS, f_R

It is obviously seen that the tolerances of the design parameters have a fatal influence to the sensitivity and the resonance behavior, solely the cross sensibility remains in the tolerable range. The probability is about 50% that the accelerometer works outside of the specified properties range. Such a behavior is typical since the optimum design is normally located on the boundary of the permissible design parameter space.

The Pareto charts allow to see the importance of the design tolerances which were included into the calculations for the functional parameters (Figure 10).

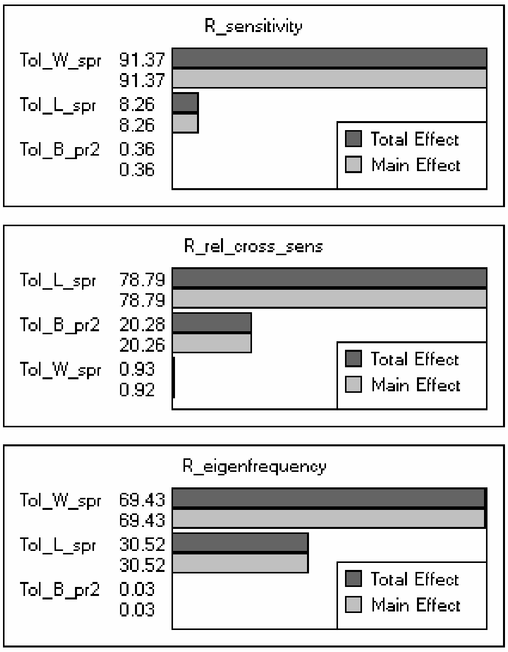

Figure 10. Pareto Charts of S, CS, f_R

For example, the spread of the sensitivity is essentially caused by the tolerance of the width of the spring W_{spr} whereas the influence of the tolerance of the distance B_{pr} is negligible. Evidently, the total effects result from the main effects alone in all of the calculated interrelations. That allows to neglect the terms resulting from the pairwise combination of the tolerances. Therefore a reduced second order analysis with only linear dependence of the computational effort on the number of the variables is sufficient for the calculation of the robust design.

In a following step, a robust design of the ceramic thick film accelerometer can be calculated. As a result we will obtain a design that is optimized for a set of functional requirements and design tolerances.

4. Conclusions

OptiY and COMSOL Multiphysics are easy to connect at the script interface. The OptiY tool allows to perform different numerical experiments, e.g. nominal optimization, tolerance analysis including sensitivity analysis and failure probability estimation, and a probability based design optimization as well, e.g. the design for robustness.

When the solution time of the COMSOL multiphysics model is about five minutes, a nominal optimization requires less than one day on a standard PC. The computational effort of the tolerance analysis increases with the squared number of the tolerances if based on a second order analysis, but linearly if based on a reduced second order analysis. Depending on the number of tolerances included in the computations this causes solution times between some minutes and one day. The optimization of the tolerances of the design for robustness requires the computational effort of the tolerance analysis to be met for every single step of the optimization process. That makes clear that application is limited by today’s computing power.

As an example the nominal design optimization, the sensitivity and the system failure analysis of a thick film accelerometer were made. In this way the conditions for a design for robustness were developed.

5. References

1. T.Q. Pham, *OptiY Software and Documentation Version 2.3*, OptiY e.K 2007 www.optiy.de
2. J.M. Browne, *Probabilistic Design*, http://grassmannalgebra.info/probabilisticdesign 2007
3. M.W. Long, J.D. Narciso, *Probabilistic Design Methodology for Composite Aircraft Structures*, Report DOT/FAA/AR-99-2, U.S. Department of Transportation, Federal Aviation Administation (1999)
4. A. Saltelli, K. Chan, E.M. Scott, *Sensitivity Analysis*. John Willey & Sons Chichester, New York (2000).
5. W. Chen, J.K. Allen, K.-L. Tsui, F. Mistree, A Procedure for Robust Design: Minimizing Variations Caused by Noise Factors and Control

Factors, *ASME Journal of Mechanical Design*. **118**, 478-485 (1996)

6. N.P. Koch, B. Wujek, O. Golovidov, A Multi-stage, parallel Implementation of Probabilistic Design Optimization in an MDO Framework, *8th Symposium on Multidisciplinary Analysis and Optimization*, September 2000, CA.

7. U. Partsch, S. Gebhardt, D. Arndt, H. Georgi, H. Neubert, D. Fleischer, M. Gruchow, LTCC-based Sensors for Mechanical Quantities. *European Microelectronics and Packaging Conference (EMPC) 2007*, Oulu (FI), June 17-20 (2007)