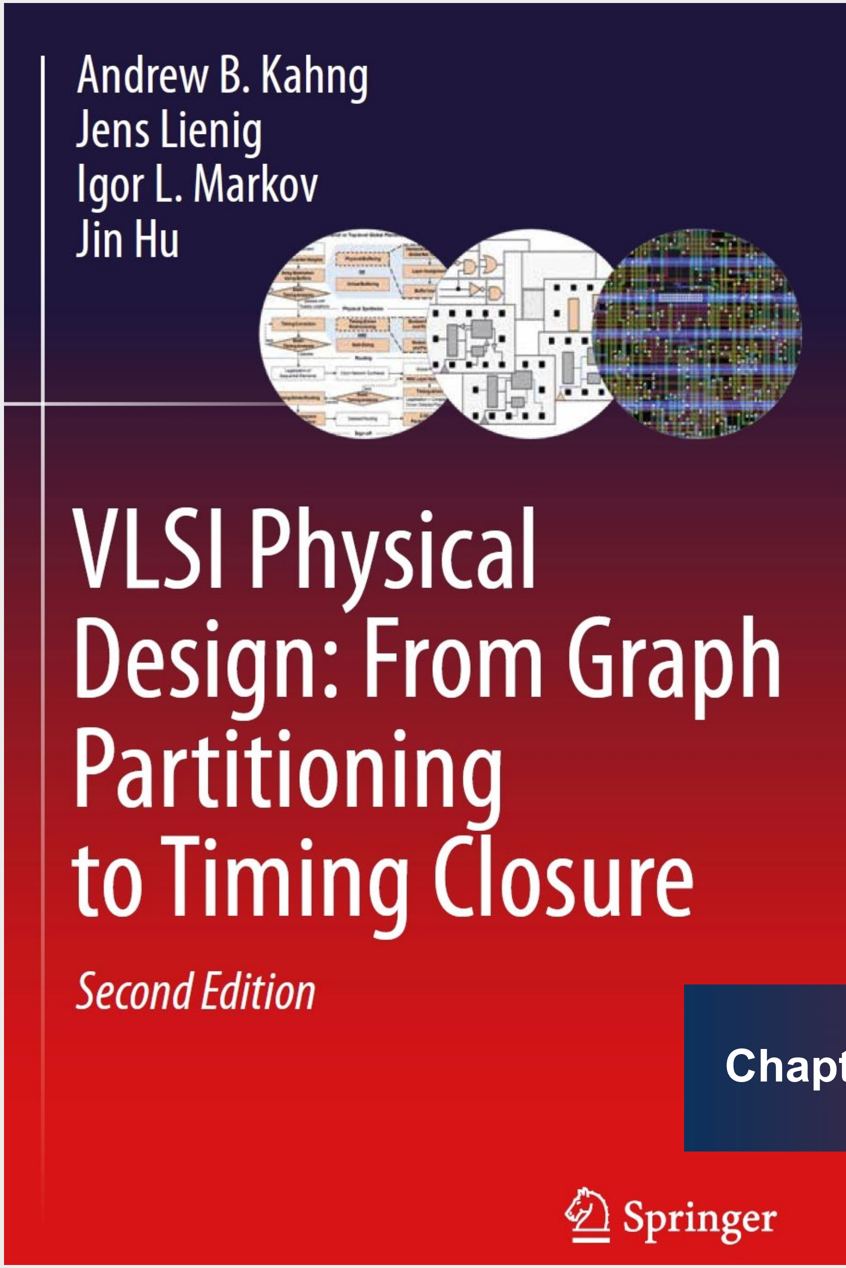
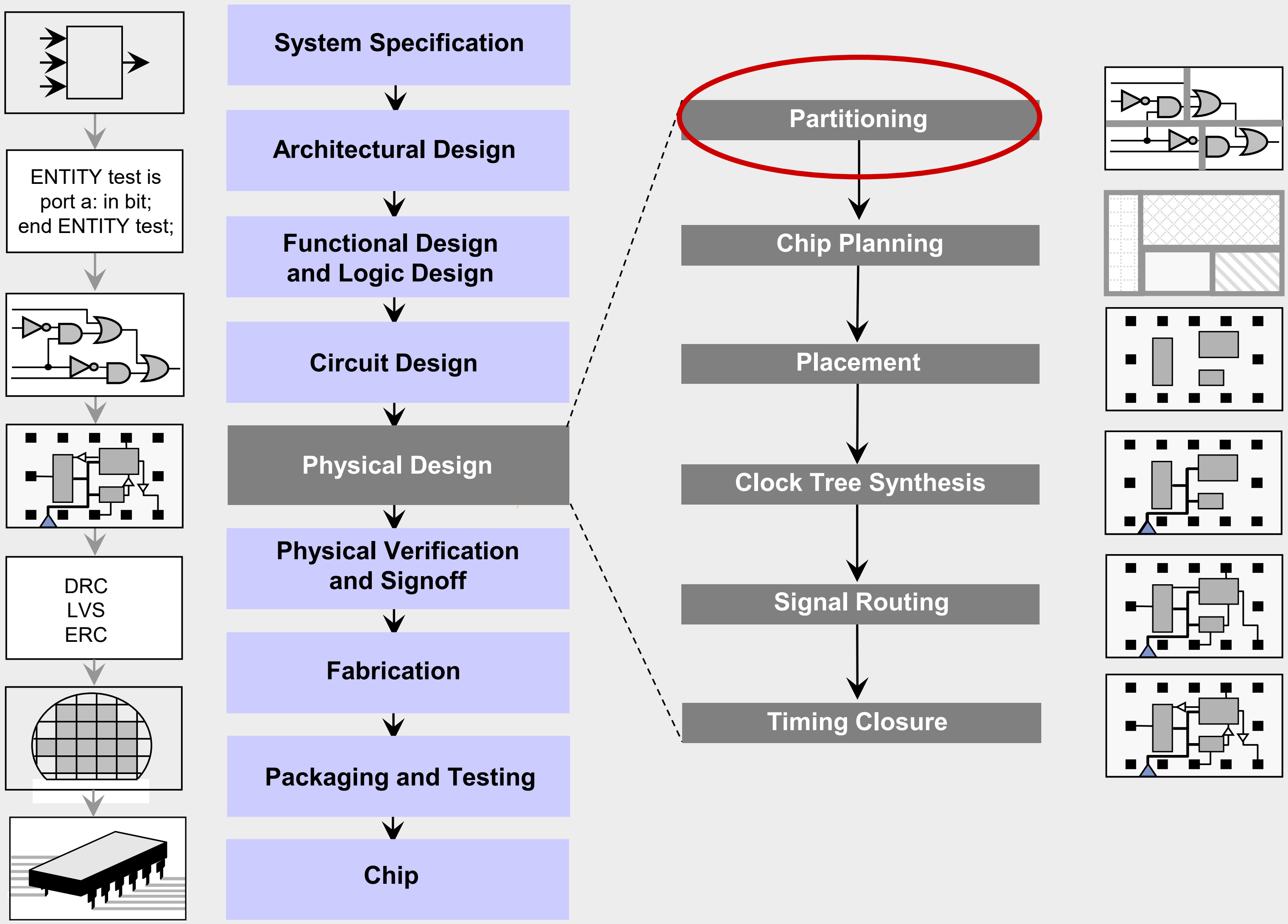
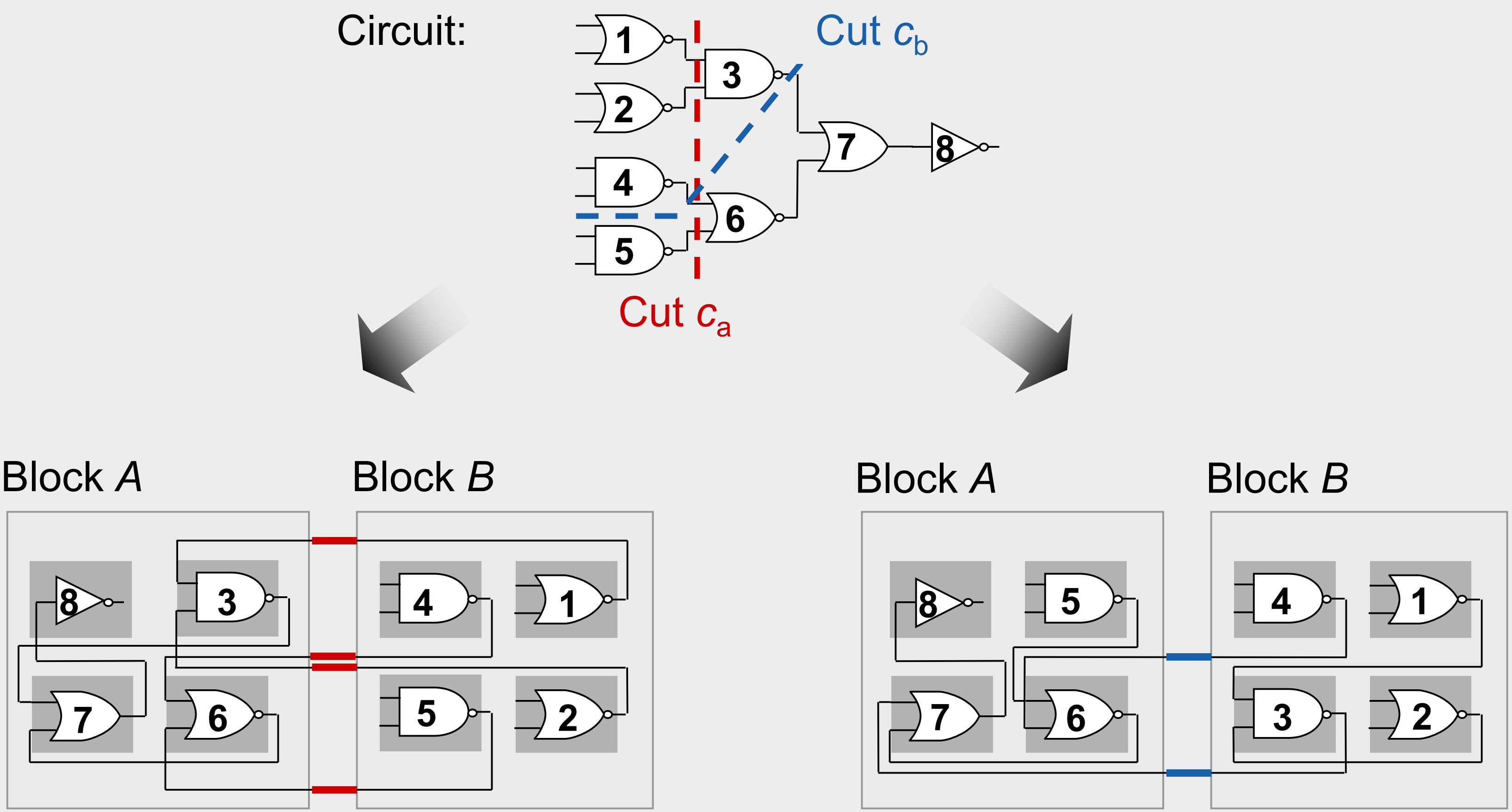


Chapter 2 – Netlist and System Partitioning

Chapter 2 – Netlist and System Partitioning

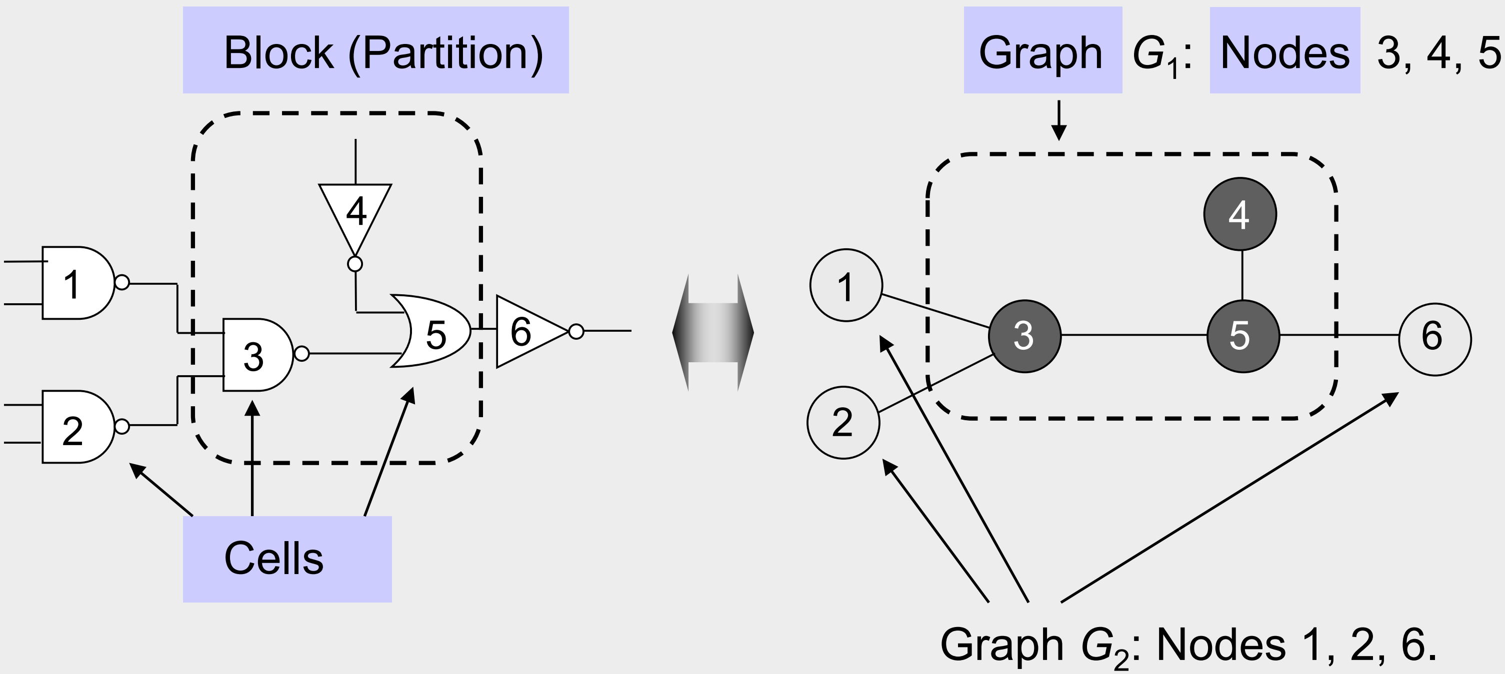

ENTITY test is
port a: in bit;
end ENTITY test;
DRC
LVS
ERC
System Specification
Architectural Design
Functional Design
and Logic Design
Circuit Design
Physical Design
Physical Verification
and Signoff
Fabrication
Packaging and Testing
Chip
Partitioning
Chip Planning
Placement
Clock Tree Synthesis
Signal Routing
Timing Closure



Cut c_a: four external connections

Cut c_b: two external connections

2.2 Terminology



Collection of cut edges

Cut set: (1,3), (2,3), (5,6),

2.3 Optimization Goals

- Given a graph $G(V,E)$ with $|V|$ nodes and $|E|$ edges where each node $v \in V$ and each edge $e \in E$.
- Each node has area $s(v)$ and each edge has cost or weight $w(e)$.
- The objective is to divide the graph G into k disjoint subgraphs such that all optimization goals are achieved and all original edge relations are respected.

2.3 Optimization Goals

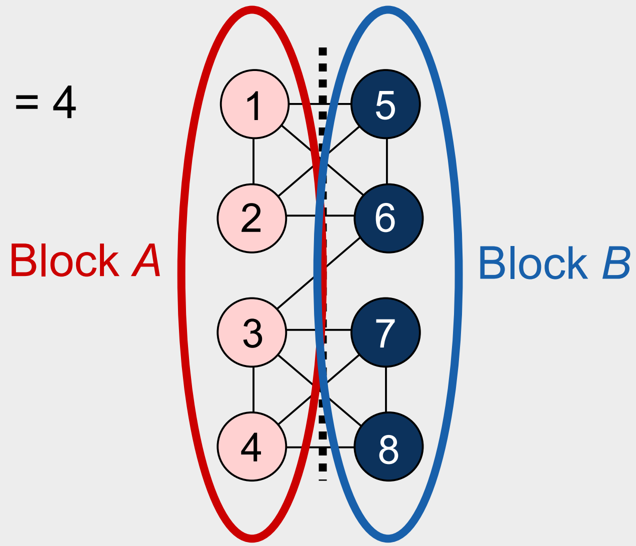
- In detail, what are the optimization goals?
  - Number of connections between partitions is minimized
  - Each partition meets all design constraints (size, number of external connections..)
  - Balance every partition as well as possible

- How can we meet these goals?
  - Unfortunately, this problem is NP-hard
  - Efficient heuristics are developed in the 1970s and 1980s.
    They are high quality and in low-order polynomial time.

Given: A graph with 2*n* nodes where each node has the same weight.

Goal: A partition (division) of the graph into two disjoint subsets *A* and *B* with minimum cut cost and $|A| = |B| = n$.

Example: $n = 4$



2.4.1 Kernighan-Lin (KL) Algorithm – Terminology

Cost $D(v)$ of moving a node v

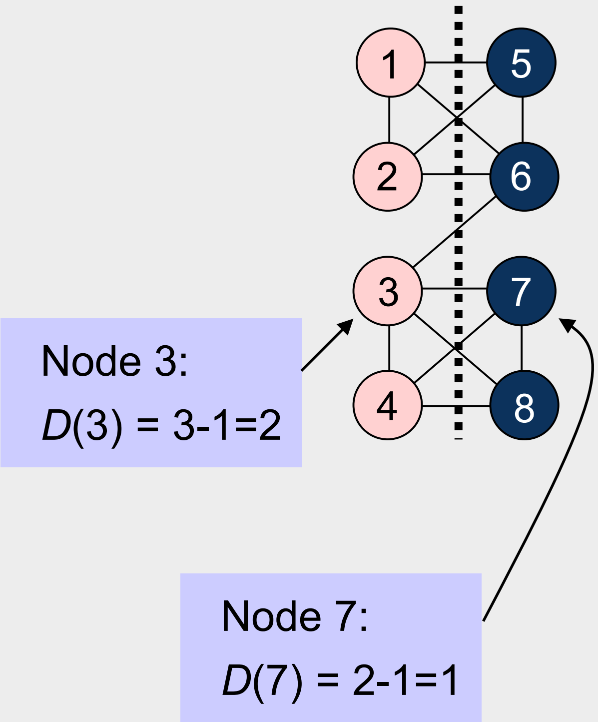
$D(v) = |E_c(v)| - |E_{nc}(v)|$,

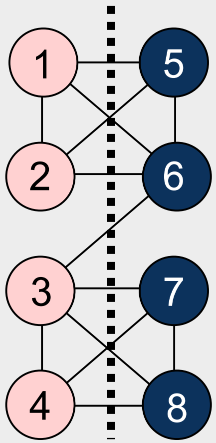
where

$E_c(v)$ is the set of v's incident edges that are cut by the cut line, and

$E_{nc}(v)$ is the set of v's incident edges that are not cut by the cut line.

High costs ($D > 0$) indicate that the node should move, while low costs ($D < 0$) indicate that the node should stay within the same partition.





Gain of swapping a pair of nodes *a* und *b*

$\Delta g = D(a) + D(b) - 2 * c(a,b),$

where

- $D(a)$, $D(b)$ are the respective costs of nodes *a*, *b*
- $c(a,b)$ is the connection weight between *a* and *b*:
  If an edge exists between *a* and *b*,
  then $c(a,b)$ = edge weight (here 1),
  otherwise, $c(a,b) = 0$.

The gain Δg indicates how useful the swap between two nodes will be

The larger Δg, the more the total cut cost will be reduced

2.4.1 Kernighan-Lin (KL) Algorithm – Terminology

Gain of swapping a pair of nodes *a* und *b*

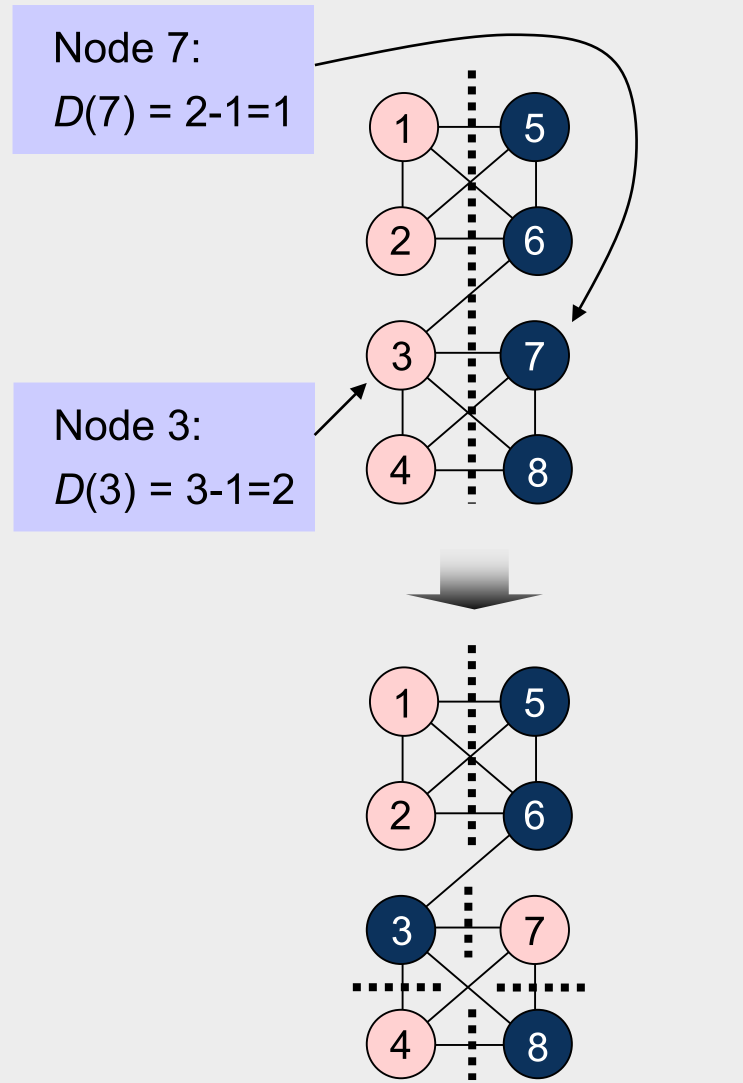
$\Delta g = D(a) + D(b) - 2 * c(a,b),$

where

- *D*(*a*), *D*(*b*) are the respective costs of nodes *a*, *b*
- *c*(*a*,*b*) is the connection weight between *a* and *b*:
  If an edge exists between *a* and *b*,
  then *c*(*a*,*b*) = edge weight (here 1),
  otherwise, *c*(*a*,*b*) = 0.

$\Delta g\ (3,7) = D(3) + D(7) - 2 * c(a,b) = 2 + 1 – 2 = 1$

=> Swapping nodes 3 and 7 would reduce the cut size by 1



2.4.1 Kernighan-Lin (KL) Algorithm – Terminology

Gain of swapping a pair of nodes *a* und *b*

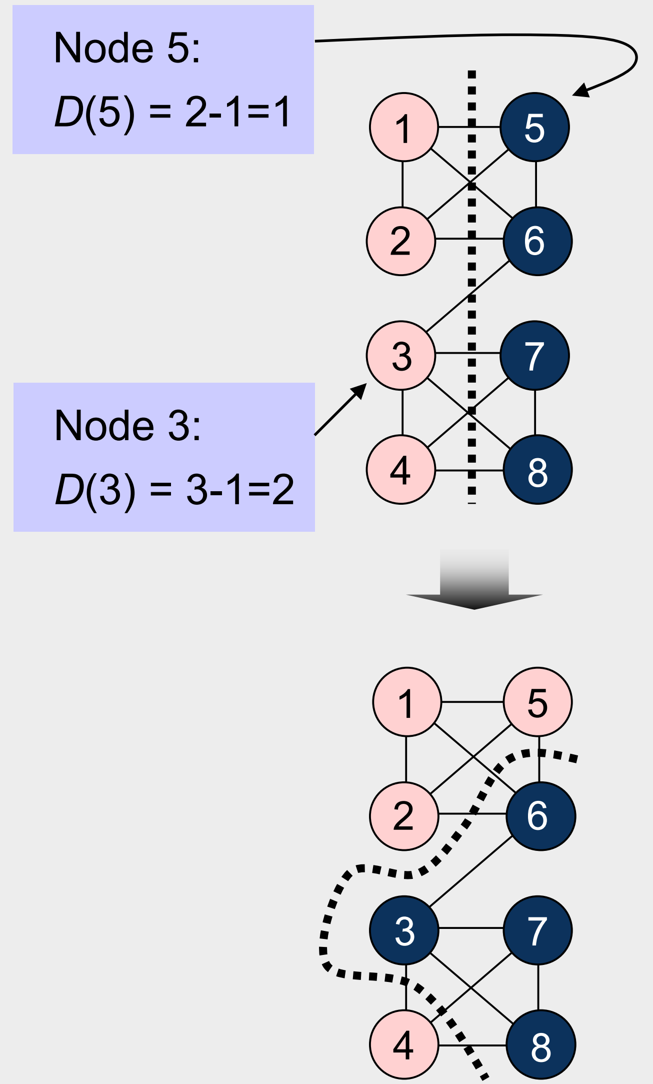
$$\Delta g = D(a) + D(b) - 2 * c(a,b),$$

where

- $D(a)$, $D(b)$ are the respective costs of nodes a, b
- $c(a,b)$ is the connection weight between a and b:
  If an edge exists between a and b,
  then $c(a,b)$ = edge weight (here 1),
  otherwise, $c(a,b) = 0$.

$$\Delta g\,(3,5) = D(3) + D(5) - 2 * c(a,b) = 2 + 1 - 0 = 3$$

=> Swapping nodes 3 and 5 would reduce the cut size by 3



2.4.1 Kernighan-Lin (KL) Algorithm – Terminology

Gain of swapping a pair of nodes *a* und *b*

The goal is to find a pair of nodes *a* and *b* to exchange such that Δg is maximized and swap them.

2.4.1 Kernighan-Lin (KL) Algorithm – Terminology

Maximum positive gain G_m of a pass

The maximum positive gain G_m corresponds to the best prefix of *m* swaps within the swap sequence of a given pass.

These *m* swaps lead to the partition with the minimum cut cost encountered during the pass.

G_m is computed as the sum of Δg values over the first *m* swaps of the pass, with *m* chosen such that G_m is maximized.

$$G_m = \sum_{i=1}^{m} \Delta g_i$$

2.4.1 Kernighan-Lin (KL) Algorithm – One pass

Step 0:
- $V = 2n$ nodes
- $\{A, B\}$ is an initial arbitrary partitioning

Step 1:
- $i = 1$
- Compute $D(v)$ for all nodes $v \in V$

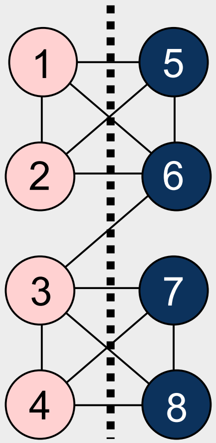
Step 2:
- Choose a_i and b_i such that $\Delta g_i = D(a_i) + D(b_i) - 2 * c(a_i b_i)$ is maximized
- Swap and fix a_i and b_i

Step 3:
- If all nodes are fixed, go to Step 4. Otherwise
- Compute and update D values for all nodes that are connected to a_i and b_i and are not fixed.
- $i = i + 1$
- Go to Step 2

Step 4:
- Find the move sequence $1 \ldots m$ ($1 \leq m \leq i$), such that $G_m = \sum_{i=1}^{m} \Delta g_i$ is maximized
- If $G_m > 0$, go to Step 5. Otherwise, END

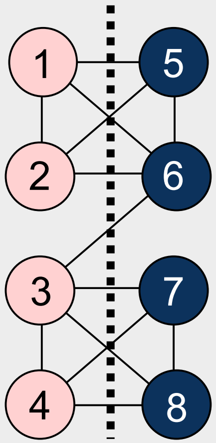
Step 5:
- Execute m swaps, reset remaining nodes
- Go to Step 1

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

2.4.1 Kernighan-Lin (KL) Algorithm – Example



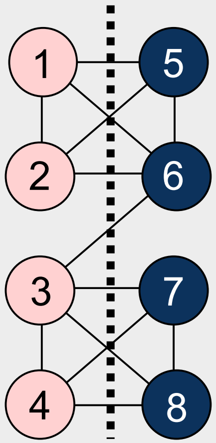
Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Costs $D(v)$ of each node:

$D(1) = 1$	$D(5) = 1$
$D(2) = 1$	$D(6) = 2$
$D(3) = 2$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

Nodes that lead to maximum gain

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Costs $D(v)$ of each node:

$D(1) = 1$	$\mathbf{D(5) = 1}$
$D(2) = 1$	$D(6) = 2$
$\mathbf{D(3) = 2}$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

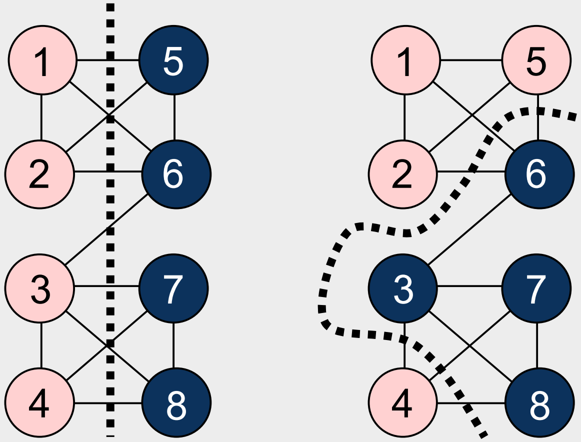
Nodes that lead to maximum gain

$\Delta g_1 = 2+1-0 = 3$ Gain after node swapping

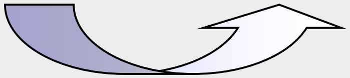
Swap (3,5)

$G_1 = \Delta g_1 = 3$ Gain in the current pass

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8



$D(1) = 1$	$\mathbf{D(5) = 1}$
$D(2) = 1$	$D(6) = 2$
$\mathbf{D(3) = 2}$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

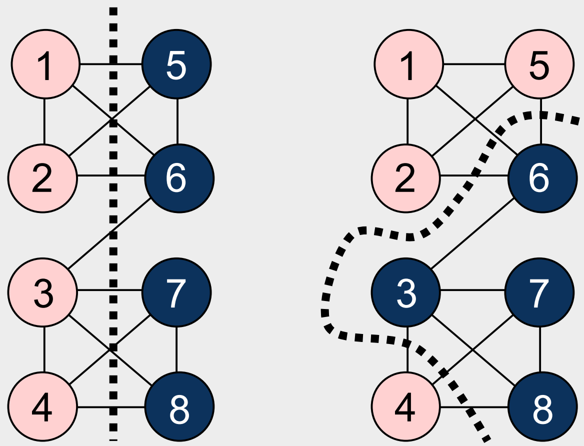
Nodes that lead to maximum gain

$\Delta g_1 = 2+1-0 = 3$ — Gain after node swapping

Swap (3,5)

$G_1 = \Delta g_1 = 3$ — Gain in the current pass

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Cut cost: 6
Not fixed:
1,2,4,6,7,8

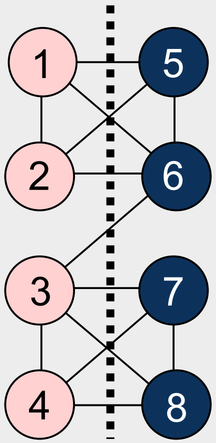
$D(1) = 1$	$\boldsymbol{D(5) = 1}$
$D(2) = 1$	$D(6) = 2$
$\boldsymbol{D(3) = 2}$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$

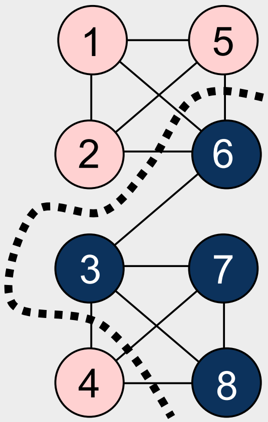
Swap (3,5)

$G_1 = \Delta g_1 = 3$

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8



Cut cost: 6
Not fixed:
1,2,4,6,7,8

$D(1) = 1$	$\mathbf{D(5) = 1}$
$D(2) = 1$	$D(6) = 2$
$\mathbf{D(3) = 2}$	$D(7) = 1$
$D(4) = 1$	$D(8) = 1$

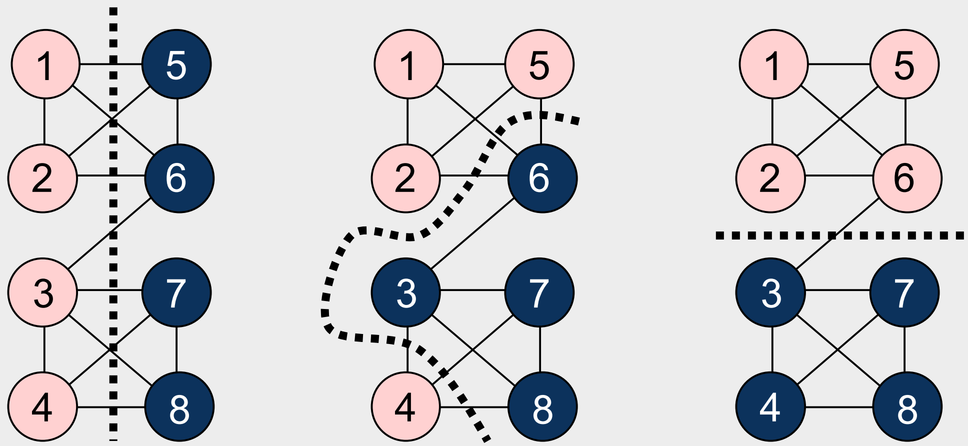
$\Delta g_1 = 2+1-0 = 3$

Swap (3,5)

$G_1 = \Delta g_1 = 3$

$D(1) = -1$	$\mathbf{D(6) = 2}$
$D(2) = -1$	$D(7)=-1$
$\mathbf{D(4) = 3}$	$D(8)=-1$

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9
Not fixed:
1,2,3,4,5,6,7,8

Cut cost: 6
Not fixed:
1,2,4,6,7,8

$D(1) = 1$ $\quad$ $\boldsymbol{D(5) = 1}$
$D(2) = 1$ $\quad$ $D(6) = 2$
$\boldsymbol{D(3) = 2}$ $\quad$ $D(7) = 1$
$D(4) = 1$ $\quad$ $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$ $\quad$ $\boldsymbol{D(6) = 2}$
$D(2) = -1$ $\quad$ $D(7)=-1$
$\boldsymbol{D(4) = 3}$ $\quad$ $D(8)=-1$

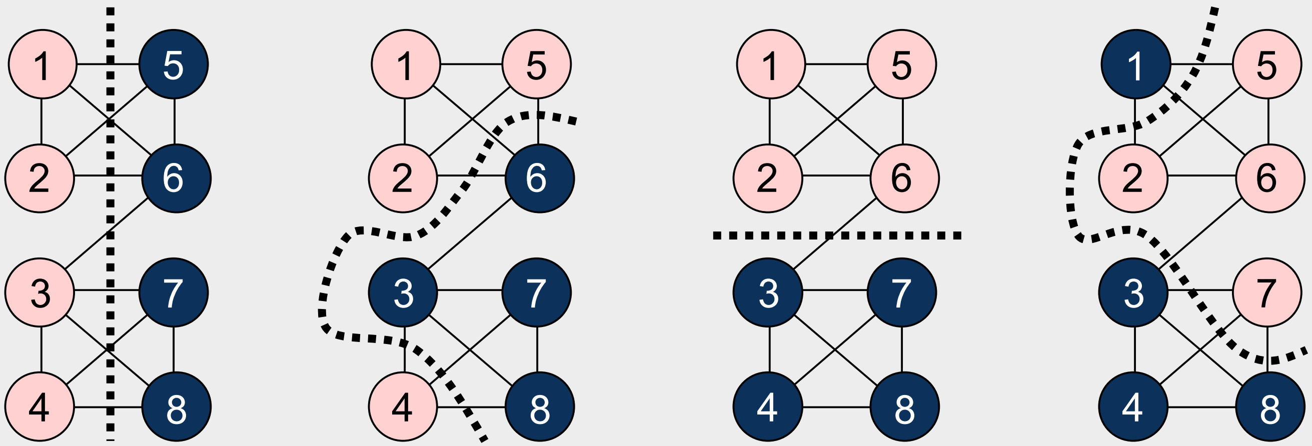
$\Delta g_2 = 3+2-0 = 5$
Swap (4,6)
$G_2 = G_1+\Delta g_2 = 8$

Nodes that lead to maximum gain

Gain after node swapping

Gain in the current pass

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9 Not fixed: 1,2,3,4,5,6,7,8	Cut cost: 6 Not fixed: 1,2,4,6,7,8	Cut cost: 1 Not fixed: 1,2,7,8	Cut cost: 7 Not fixed: 2,8

Step 1

$D(1) = 1$, $D(2) = 1$, $\boldsymbol{D(3) = 2}$, $D(4) = 1$

$\boldsymbol{D(5) = 1}$, $D(6) = 2$, $D(7) = 1$, $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$

Swap (3,5)

$G_1 = \Delta g_1 = 3$

Step 2

$D(1) = -1$, $D(2) = -1$, $\boldsymbol{D(4) = 3}$

$\boldsymbol{D(6) = 2}$, $D(7) = -1$, $D(8) = -1$

$\Delta g_2 = 3+2-0 = 5$

Swap (4,6)

$G_2 = G_1 + \Delta g_2 = 8$

Step 3

$\boldsymbol{D(1) = -3}$, $D(2) = -3$

$\boldsymbol{D(7) = -3}$, $D(8) = -3$

$\Delta g_3 = -3-3-0 = -6$

Swap (1,7)

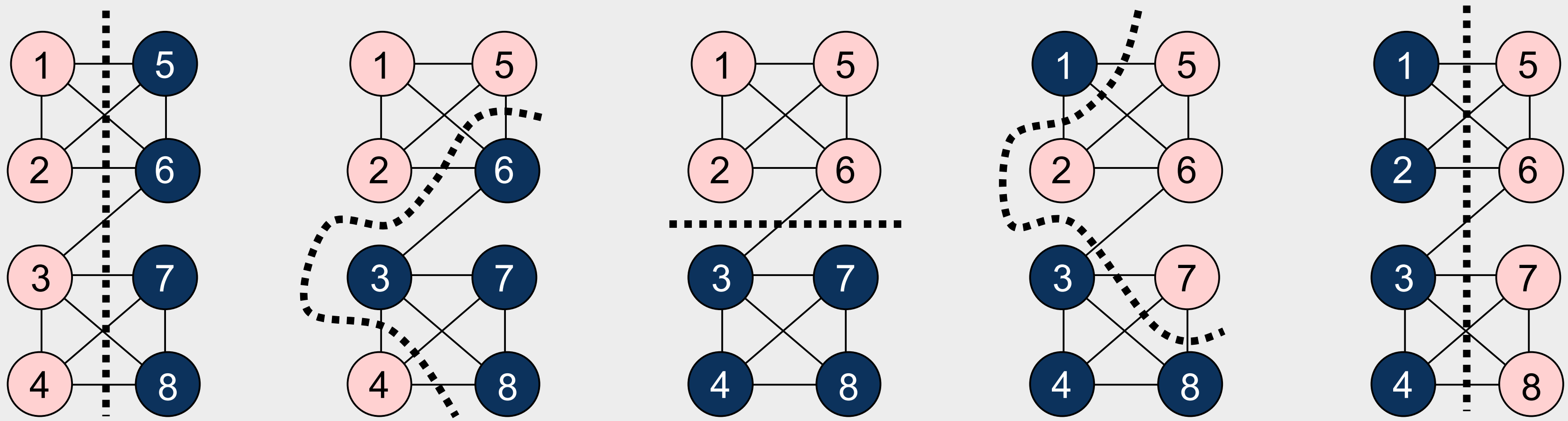
$G_3 = G_2 + \Delta g_3 = 2$

Nodes that lead to maximum gain

Gain after node swapping

Gain in the current pass

2.4.1 Kernighan-Lin (KL) Algorithm – Example



Cut cost: 9 Not fixed: 1,2,3,4,5,6,7,8	Cut cost: 6 Not fixed: 1,2,4,6,7,8	Cut cost: 1 Not fixed: 1,2,7,8	Cut cost: 7 Not fixed: 2,8	Cut cost: 9 Not fixed: –

$D(1) = 1$ **$D(5) = 1$**
$D(2) = 1$ $D(6) = 2$
$D(3) = 2$ $D(7) = 1$
$D(4) = 1$ $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$ **$D(6) = 2$**
$D(2) = -1$ $D(7)=-1$
$D(4) = 3$ $D(8)=-1$

$\Delta g_2 = 3+2-0 = 5$
Swap (4,6)
$G_2 = G_1 + \Delta g_2 = 8$

$D(1) = -3$ **$D(7)=-3$**
$D(2) = -3$ $D(8)=-3$

$\Delta g_3 = -3-3-0 = -6$
Swap (1,7)
$G_3 = G_2 + \Delta g_3 = 2$

$D(2) = -1$ **$D(8)=-1$**

$\Delta g_4 = -1-1-0 = -2$
Swap (2,8)
$G_4 = G_3 + \Delta g_4 = 0$

2.4.1 Kernighan-Lin (KL) Algorithm – Example

$D(1) = 1$ **$D(5) = 1$**
$D(2) = 1$ $D(6) = 2$
$D(3) = 2$ $D(7) = 1$
$D(4) = 1$ $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$ **$D(6) = 2$**
$D(2) = -1$ $D(7)=-1$
$D(4) = 3$ $D(8)=-1$

$\Delta g_2 = 3+2-0 = 5$
Swap (4,6)
$G_2 = G_1+\Delta g_2 = 8$

$D(1) = -3$ **$D(7)=-3$**
$D(2) = -3$ $D(8)=-3$

$\Delta g_3 = -3-3-0 = -6$
Swap (1,7)
$G_3 = G_2 + \Delta g_3 = 2$

$D(2) = -1$ **$D(8)=-1$**

$\Delta g_4 = -1-1-0 = -2$
Swap (2,8)
$G_4 = G_3 + \Delta g_4 = 0$

Maximum positive gain $G_m = 8$ with $m = 2$.

2.4.1 Kernighan-Lin (KL) Algorithm – Example

$D(1) = 1$ **$D(5) = 1$**
$D(2) = 1$ $D(6) = 2$
$D(3) = 2$ $D(7) = 1$
$D(4) = 1$ $D(8) = 1$

$\Delta g_1 = 2+1-0 = 3$
Swap (3,5)
$G_1 = \Delta g_1 = 3$

$D(1) = -1$ **$D(6) = 2$**
$D(2) = -1$ $D(7) = -1$
$D(4) = 3$ $D(8) = -1$

$\Delta g_2 = 3+2-0 = 5$
Swap (4,6)
$G_2 = G_1 + \Delta g_2 = 8$

$D(1) = -3$ **$D(7) = -3$**
$D(2) = -3$ $D(8) = -3$

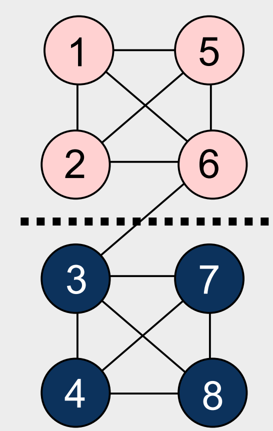
$\Delta g_3 = -3-3-0 = -6$
Swap (1,7)
$G_3 = G_2 + \Delta g_3 = 2$

$D(2) = -1$ **$D(8) = -1$**

$\Delta g_4 = -1-1-0 = -2$
Swap (2,8)
$G_4 = G_3 + \Delta g_4 = 0$

Maximum positive gain $G_m = 8$ with $m = 2$.

Since $G_m > 0$, the first $m = 2$ swaps (3,5) and (4,6) are executed.



Since $G_m > 0$, more passes are needed until $G_m \leq 0$.

2.4.2 Extensions of the Kernighan-Lin (KL) Algorithm

- Unequal partition sizes
  - Apply the KL algorithm with only min(|*A*|,|*B*|) pairs swapped

- Unequal node weights
  - Try to rescale weights to integers, e.g., as multiples of *the greatest common divisor* of all node weights
  - Maintain area balance or allow a *one-move deviation* from balance

- *k*-way partitioning (generating *k* partitions)
  - Apply the KL two-way partitioning algorithm to all possible pairs of partitions
  - Recursive partitioning (convenient when k is a power of two)
  - Direct k-way extensions exist

2.4.3 Fiduccia-Mattheyses (FM) Algorithm

- Single cells are moved independently instead of swapping pairs of cells --- cannot and do not need to maintain exact partition balance
  - The area of each individual cell is taken into account
  - Applicable to partitions of unequal size and in the presence of initially fixed cells
- Cut costs are extended to include hypergraphs
  - nets with 2+ pins
- While the KL algorithm aims to minimize cut costs based on edges, the FM algorithm minimizes cut costs based on nets
- Nodes and subgraphs are referred to as *cells* and *blocks*, respectively

2.4.3 Fiduccia-Mattheyses (FM) Algorithm

Given: a hypergraph $G(V,H)$ with nodes and *weighted* hyperedges
partition size constraints

Goal: to assign all nodes to disjoint partitions,
so as to minimize the total cost (weight) of all cut nets
while satisfying *partition size constraints*

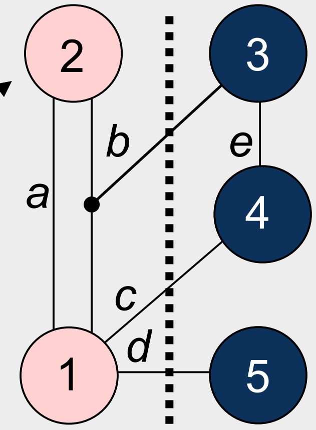
Gain $\Delta g(c)$ for cell c

$\Delta g(c) = FS(c) - TE(c)$,

where

the "moving force" $FS(c)$ is the number of nets connected to c but not connected to any other cells within c's partition, i.e., cut nets that connect only to c, and

the "retention force" $TE(c)$ is the number of *uncut* nets connected to c.



Cell 2: $FS(2) = 0$ $TE(2) = 1$ $\Delta g(2) = -1$

The higher the gain $\Delta g(c)$, the higher is the priority to move the cell c to the other partition.

Gain $\Delta g(c)$ for cell c

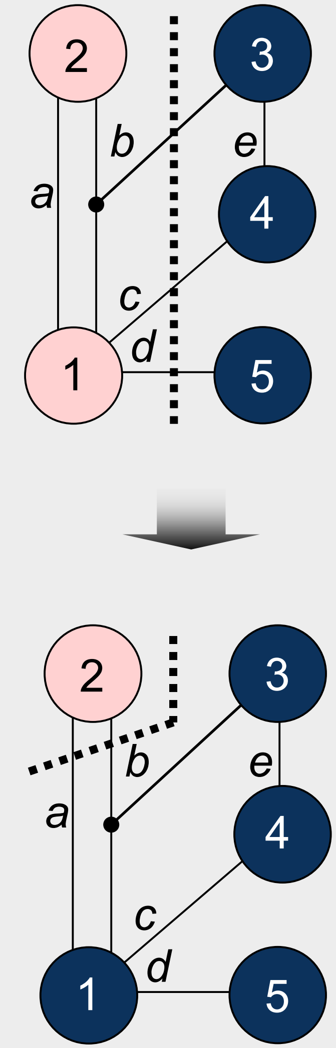
$$\Delta g(c) = FS(c) - TE(c) ,$$

where

the "moving force" $FS(c)$ is the number of nets connected to c but not connected to any other cells within c's partition, i.e., cut nets that connect only to c, and

the "retention force" $TE(c)$ is the number of *uncut* nets connected to c.

Cell 1:	$FS(1) = 2$	$TE(1) = 1$	$\Delta g(1) = 1$
Cell 2:	$FS(2) = 0$	$TE(2) = 1$	$\Delta g(2) = -1$
Cell 3:	$FS(3) = 1$	$TE(3) = 1$	$\Delta g(3) = 0$
Cell 4:	$FS(4) = 1$	$TE(4) = 1$	$\Delta g(4) = 0$
Cell 5:	$FS(5) = 1$	$TE(5) = 0$	$\Delta g(5) = 1$



Maximum positive gain G_m of a pass

The maximum positive gain G_m is the cumulative cell gain of m moves that produce a minimum cut cost.

G_m is determined by the maximum sum of cell gains Δg over a prefix of m moves in a pass

$$G_m = \sum_{i=1}^{m} \Delta g_i$$

2.4.3 Fiduccia-Mattheyses (FM) Algorithm – Terminology

Ratio factor

The *ratio factor* is the relative balance between the two partitions with respect to cell area

It is used to prevent all cells from clustering into one partition.

The ratio factor r is defined as

$$r = \frac{area(A)}{area(A) + area(B)}$$

where *area*(*A*) and *area*(*B*) are the total respective areas of partitions *A* and *B*

2.4.3 Fiducciа-Mattheyses (FM) Algorithm – Terminology

Balance criterion

The balance criterion enforces the ratio factor.

To ensure feasibility, the maximum cell area $area_{max}(V)$ must be taken into account.

A partitioning of V into two partitions A and B is said to be balanced if

$$[\ r \cdot area(V) - area_{max}(V)\ ] \leq area(A) \leq [\ r \cdot area(V) + area_{max}(V)\ ]$$

Base cell

A base cell is a cell c that has the greatest cell gain $\Delta g(c)$ among all free cells, and whose move does not violate the balance criterion.

Base cell

Cell 1:	$FS(1) = 2$	$TE(1) = 1$	$\Delta g(1) = 1$
Cell 2:	$FS(2) = 0$	$TE(2) = 1$	$\Delta g(2) = -1$
Cell 3:	$FS(3) = 1$	$TE(3) = 1$	$\Delta g(3) = 0$
Cell 4:	$FS(4) = 1$	$TE(4) = 1$	$\Delta g(4) = 0$

2.4.3 Fiduccia-Mattheyses (FM) Algorithm - One pass

Step 0: Compute the balance criterion

Step 1: Compute the cell gain Δg_1 of each cell

Step 2: $i = 1$

– Choose base cell c_1 that has maximal gain Δg_1 , move this cell

Step 3:

– Fix the base cell c_i

– Update all cells' gains that are connected to critical nets via the base cell c_i

Step 4:

– If all cells are fixed, go to Step 5. If not:

– Choose next base cell c_i with maximal gain Δg_i and move this cell

– $i = i + 1$, go to Step 3

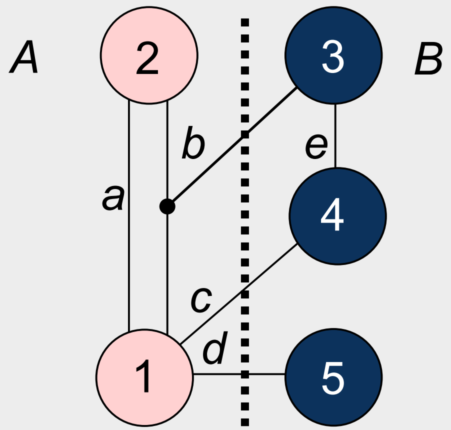
Step 5:

– Determine the best move sequence $c_1, c_2, .., c_m$ $(1 \leq m \leq i)$, so that $G_m = \sum_{i=1}^{m} \Delta g_i$ is maximized

– If $G_m > 0$, go to Step 6. Otherwise, END

Step 6:

2.4.3 Fiduccia-Mattheyses (FM) Algorithm – Example

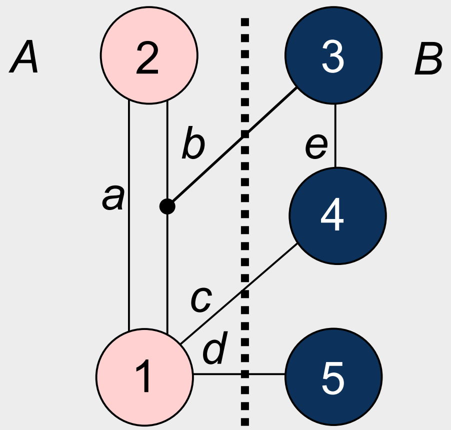


Given:
Ratio factor r = 0,375
$area$(Cell_1) = 2
$area$(Cell_2) = 4
$area$(Cell_3) = 1
$area$(Cell_4) = 4
$area$(Cell_5) = 5.

Step 0: Compute the balance criterion

$$[\, r \cdot area(V) - area_{max}(V) \,] \leq area(A) \leq [\, r \cdot area(V) + area_{max}(V) \,]$$

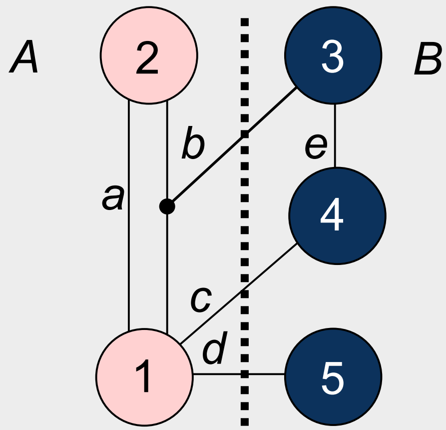
$$0{,}375 * 16 - 5 = 1 \leq area(A) \leq 11 = 0{,}375 * 16 + 5.$$



Step 1: Compute the gains of each cell

Cell 1:	*FS*(Cell_1) = 2	*TE*(Cell_1) = 1	Δg(Cell_1) = 1
Cell 2:	*FS*(Cell_2) = 0	*TE*(Cell_2) = 1	Δg(Cell_2) = -1
Cell 3:	*FS*(Cell_3) = 1	*TE*(Cell_3) = 1	Δg(Cell_3) = 0
Cell 4:	*FS*(Cell_4) = 1	*TE*(Cell_4) = 1	Δg(Cell_4) = 0
Cell 5:	*FS*(Cell_5) = 1	*TE*(Cell_5) = 0	Δg(Cell_5) = 1

2.4.3 Fiduccia-Mattheyses (FM) Algorithm – Example



Cell1:	$FS(\text{Cell\_1}) = 2$	$TE(\text{Cell\_1}) = 1$	$\Delta g(\text{Cell\_1}) = 1$
Cell 2:	$FS(\text{Cell\_2}) = 0$	$TE(\text{Cell\_2}) = 1$	$\Delta g(\text{Cell\_2}) = -1$
Cell 3:	$FS(\text{Cell\_3}) = 1$	$TE(\text{Cell\_3}) = 1$	$\Delta g(\text{Cell\_3}) = 0$
Cell 4:	$FS(\text{Cell\_4}) = 1$	$TE(\text{Cell\_4}) = 1$	$\Delta g(\text{Cell\_4}) = 0$
Cell 5:	$FS(\text{Cell\_5}) = 1$	$TE(\text{Cell\_5}) = 0$	$\Delta g(\text{Cell\_5}) = 1$

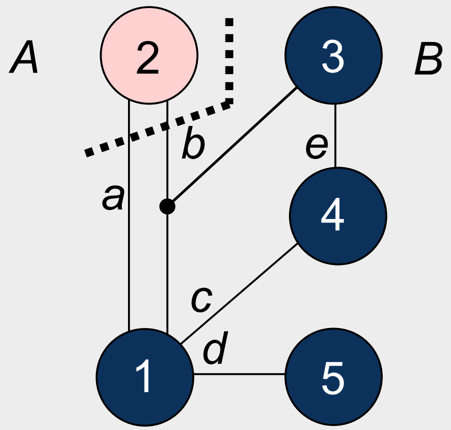
Step 2: Select the base cell

Possible base cells are Cell 1 and Cell 5

Balance criterion after moving Cell 1: $area(A) = area(\text{Cell\_2}) = 4$

Balance criterion after moving Cell 5: $area(A) = area(Cell\_1) + area(Cell\_2) + area(Cell\_5) = 11$

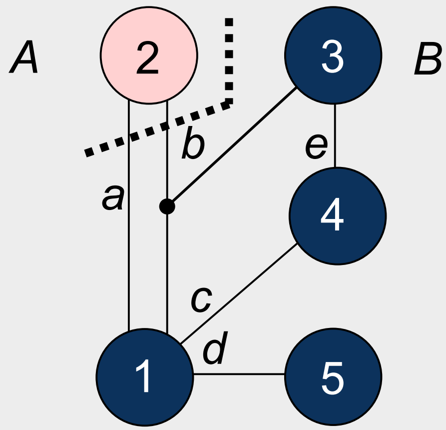
Both moves respect the balance criterion, but Cell 1 is selected, moved, and fixed as a result of the tie-breaking criterion.



Step 3: Fix base cell, update Δg values

Cell 2:	FS(Cell_2) = 2	TE(Cell_2) = 0	Δg(Cell_2) = 2
Cell 3:	FS(Cell_3) = 0	TE(Cell_3) = 1	Δg(Cell_3) = -1
Cell 4:	FS(Cell_4) = 0	TE(Cell_4) = 2	Δg(Cell_4) = -2
Cell 5:	FS(Cell_5) = 0	TE(Cell_5) = 1	Δg(Cell_5) = -1

After Iteration i = 1: Partition $A_1 = \{2\}$, Partition $B_1 = \{1,3,4,5\}$, with fixed cell $\{1\}$.



Iteration i = 1

Cell 2:	FS(Cell_2) = 2	TE(Cell_2) = 0	Δg(Cell_2) = 2
Cell 3:	FS(Cell_3) = 0	TE(Cell_3) = 1	Δg(Cell_3) = -1
Cell 4:	FS(Cell_4) = 0	TE(Cell_4) = 2	Δg(Cell_4) = -2
Cell 5:	FS(Cell_5) = 0	TE(Cell_5) = 1	Δg(Cell_5) = -1

Iteration i = 2

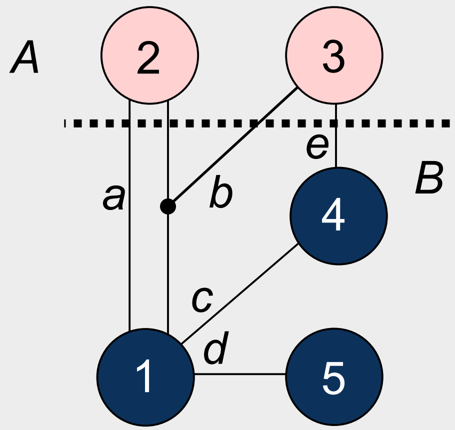
Cell *2* has maximum gain Δg_2 = 2, *area*(*A*) = 0, balance criterion is violated.

Cell *3* has next maximum gain Δg_2 = -1, *area*(*A*) = 5, balance criterion is met.

Cell *5* has next maximum gain Δg_2= -1, *area*(*A*) = 9, balance criterion is met.

Move cell *3*, updated partitions: A_2 = {2,3}, B_2 = {1,4,5}, with fixed cells {1,3}

2.4.3 Fiduccia-Mattheyses (FM) Algorithm – Example



Iteration i = 2

Cell 2: $\Delta g(\text{Cell\_2}) = 1$

Cell 4: $\Delta g(\text{Cell\_4}) = 0$

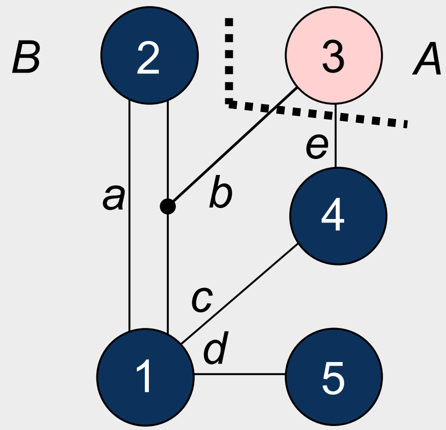
Cell 5: $\Delta g(\text{Cell\_5}) = -1$

Iteration i = 3

Cell *2* has maximum gain $\Delta g_3 = 1$, $area(A) = 1$, balance criterion is met.

Move cell 2, updated partitions: $A_3 = \{3\}$, $B_3 = \{1,2,4,5\}$, with fixed cells {1,2,3}

2.4.3 Fiduccia-Mattheyses (FM) Algorithm – Example



Iteration i = 3

Cell 4: $\Delta g(\text{Cell\_4}) = 0$

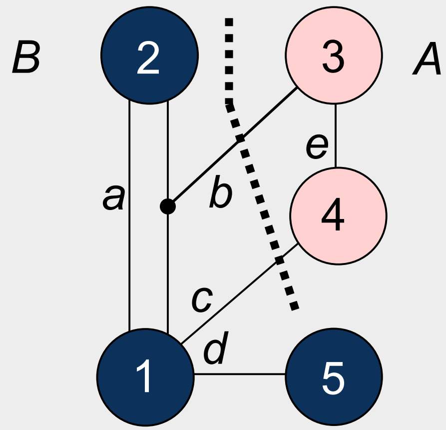
Cell 5: $\Delta g(\text{Cell\_5}) = -1$

Iteration i = 4

Cell 4 has maximum gain $\Delta g_4 = 0$, $area(A) = 5$, balance criterion is met.

Move cell 4, updated partitions: $A_4 = \{3,4\}$, $B_3 = \{1,2,5\}$, with fixed cells $\{1,2,3,4\}$

2.4.3 Fiduccia-Mattheyses (FM) Algorithm – Example



Iteration i = 4

Cell 5: $\Delta g(\text{Cell\_5}) = -1$

Iteration i = 5

Cell 5 has maximum gain $\Delta g_5 = -1$, $area(A) = 10$, balance criterion is met.

Move cell 5, updated partitions: $A_4 = \{3,4,5\}$, $B_3 = \{1,2\}$, all cells {1,2,3,4,5} fixed.

Step 5: Find best move sequence $c_1 \ldots c_m$

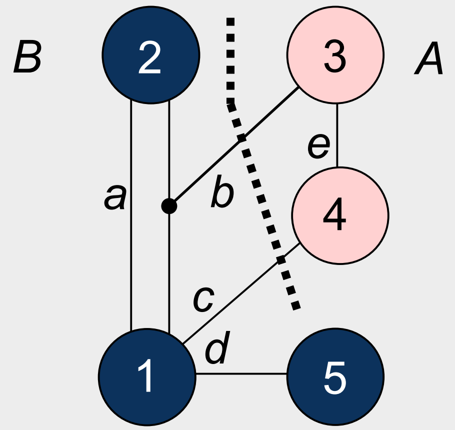
$G_1 = \Delta g_1 = 1$

$G_2 = \Delta g_1 + \Delta g_2 = 0$

$G_3 = \Delta g_1 + \Delta g_2 + \Delta g_3 = 1$

$G_4 = \Delta g_1 + \Delta g_2 + \Delta g_3 + \Delta g_4 = 1$

$G_5 = \Delta g_1 + \Delta g_2 + \Delta g_3 + \Delta g_4 + \Delta g_5 = 0.$



Maximum positive cumulative gain $G_m = \sum_{i=1}^{m} \Delta g_i = 1$

found in iterations 1, 3 and 4.

The move prefix $m = 4$ is selected due to the better balance ratio ($area(A) = 5$); the four cells 1, 2, 3 and 4 are then moved.

Result of Pass 1: Current partitions: A = {3,4}, B = {*1,2,5*}, cut cost reduced from 3 to 2.

Runtime difference between KL & FM

- Runtime of partitioning algorithms
  - KL is sensitive to the number of nodes and edges
  - FM is sensitive to the number of nodes and nets (hyperedges)
- Asymptotic complexity of partitioning algorithms
  - KL has cubic time complexity *per pass*
  - FM has linear time complexity *per pass*

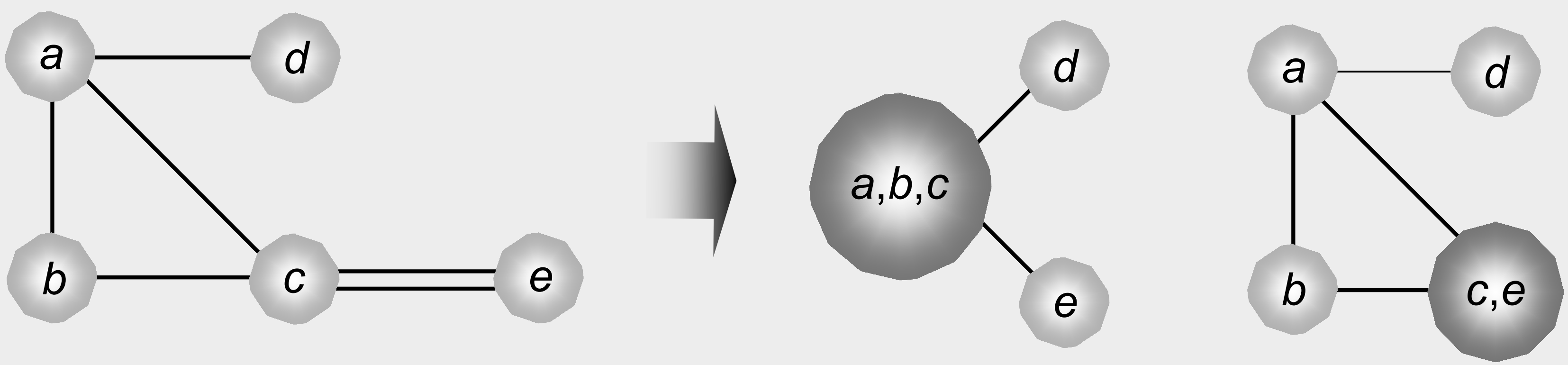
Chapter 2 – Netlist and System Partitioning

2.5.1 Clustering

- To simplify the problem, groups of tightly-connected nodes can be clustered, absorbing connections between these nodes

- Size of each cluster is often limited so as to prevent degenerate clustering, i.e. a single large cluster dominates other clusters

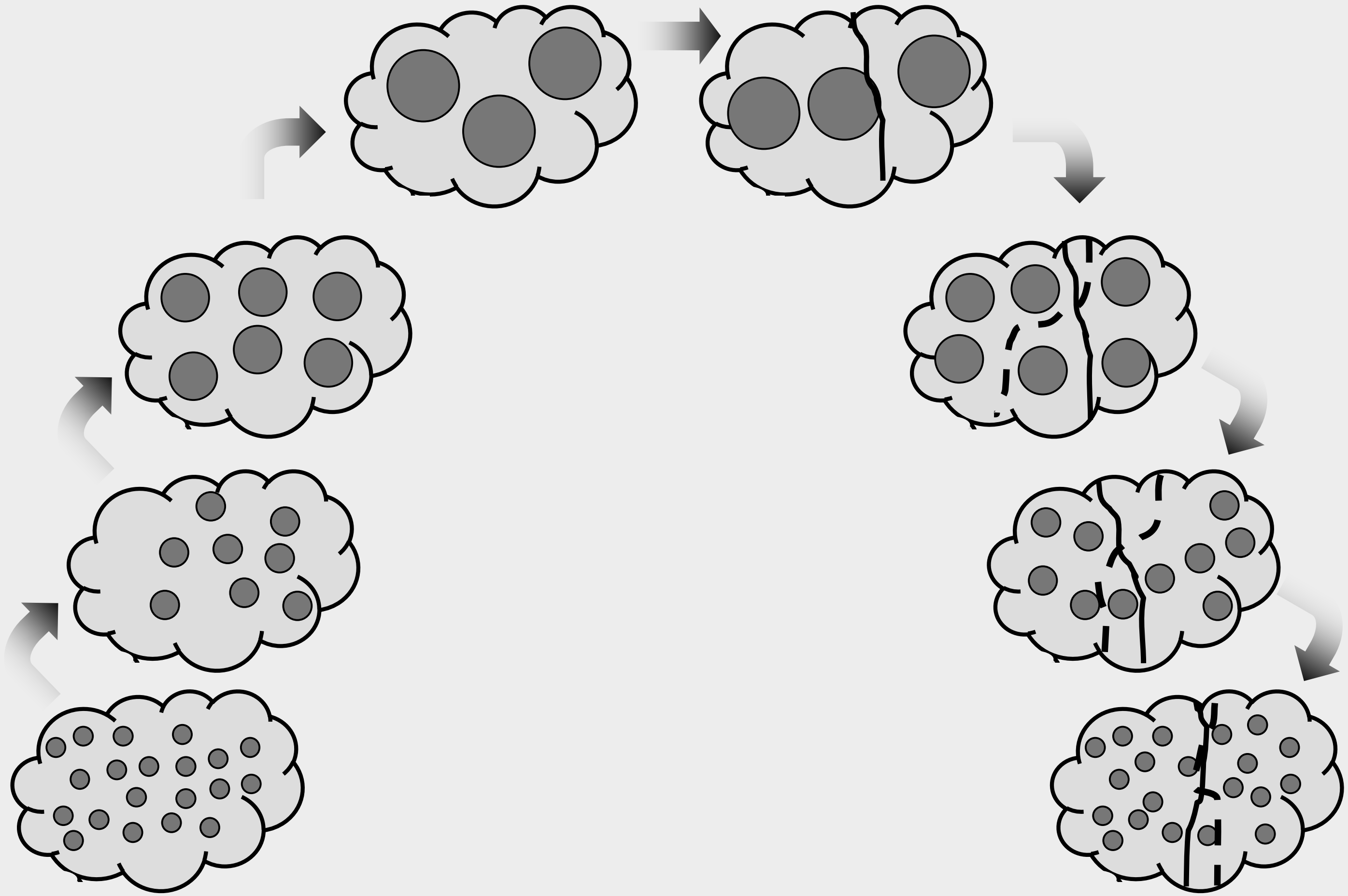
- Refinement should satisfy balance criteria



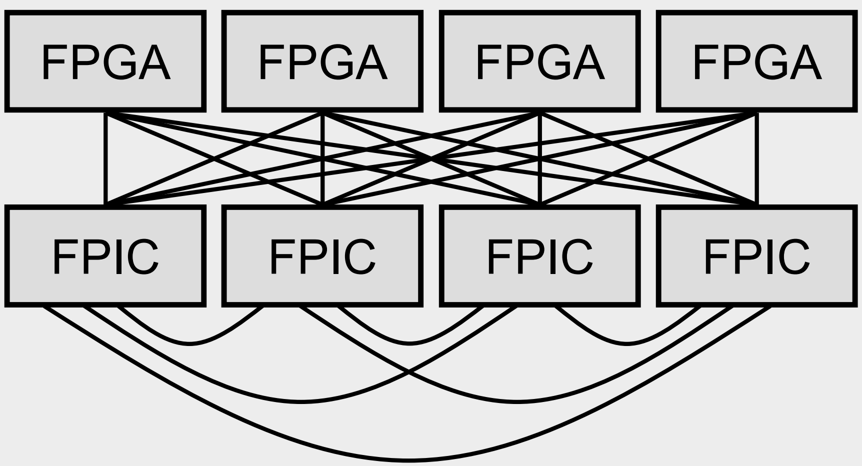
Initital graph

Possible clustering hierarchies of the graph

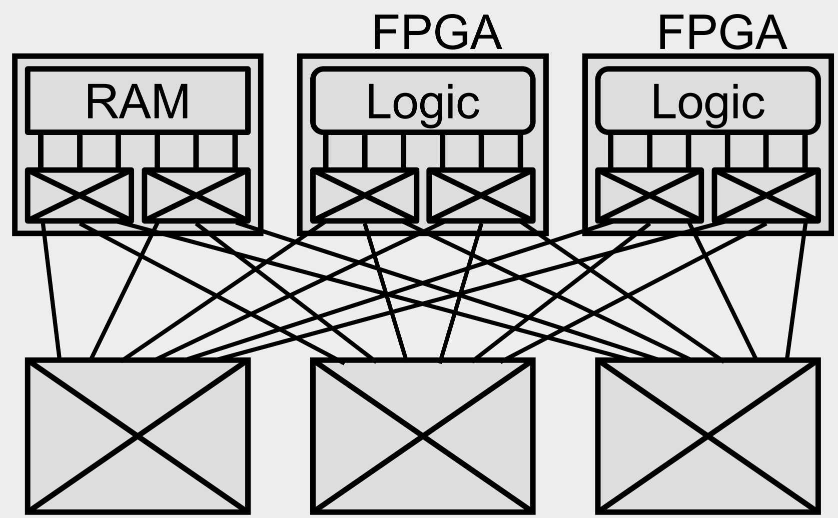
2.5.2 Multilevel Partitioning



2.6 System Partitioning onto Multiple FPGAs



Reconfigurable system with multiple FPGA and FPIC devices



Mapping of a typical system architecture onto multiple FPGAs

Summary of Chapter 2

- Circuit netlists can be represented by graphs
- Partitioning a graph means assigning nodes to disjoint partitions
  - Total size of each partition (number/area of nodes) is limited
  - Objective: minimize the number connections between partitions
- Basic partitioning algorithms
  - Move-based, move are organized into passes
  - KL swaps pairs of nodes from different partitions
  - FM re-assigns one node at a time
  - FM is faster, usually more successful
- Multilevel partitioning
  - Clustering
  - FM partitioning
  - Refinement (also uses FM partitioning)
- Application: system partitioning into FPGAs
  - Each FPGA is represented by a partition