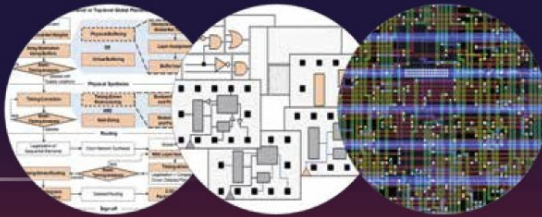


Andrew B. Kahng  
Jens Lienig  
Igor L. Markov  
Jin Hu



# VLSI Physical Design: From Graph Partitioning to Timing Closure

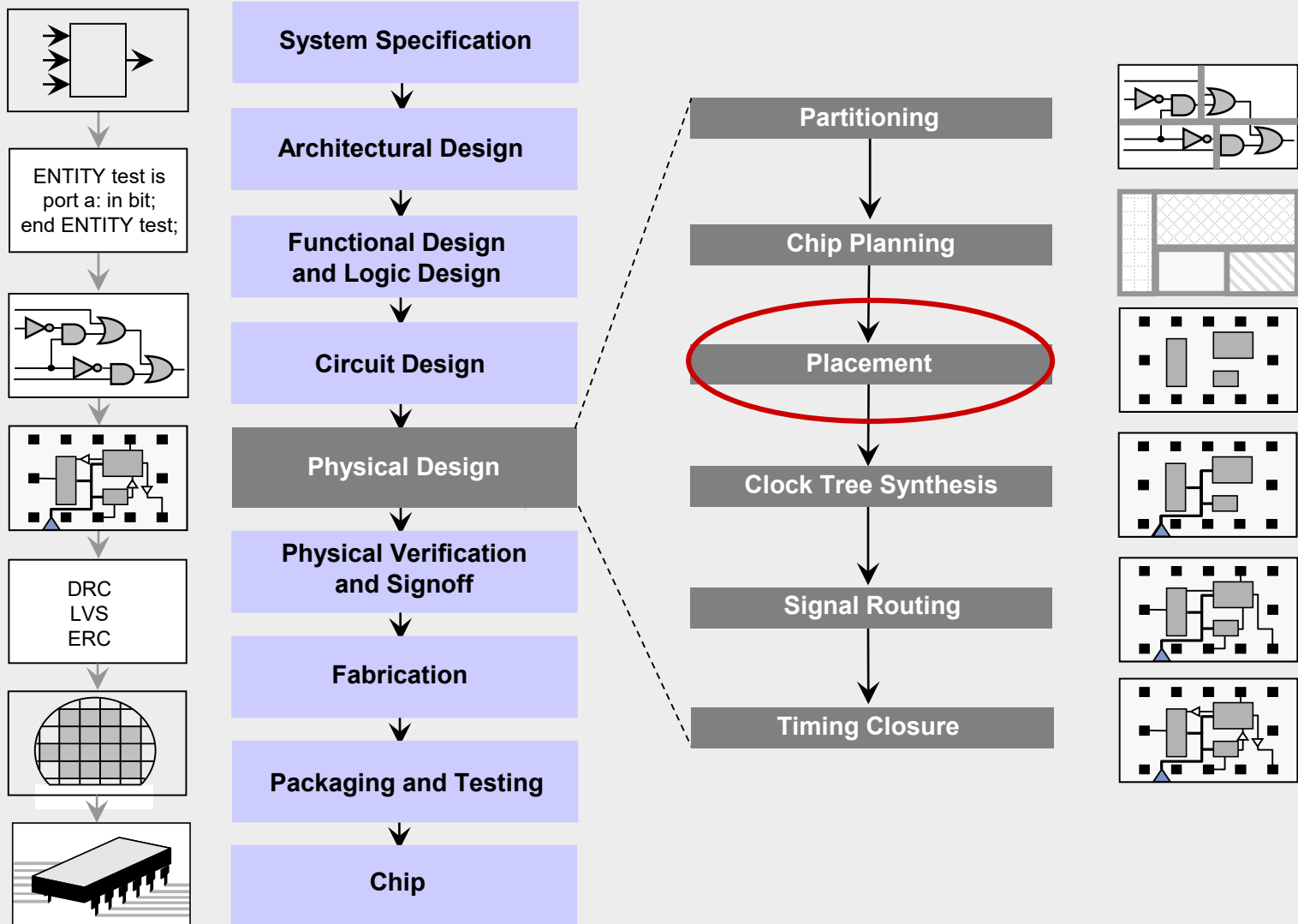
*Second Edition*

**Chapter 4 – Global and Detailed Placement**

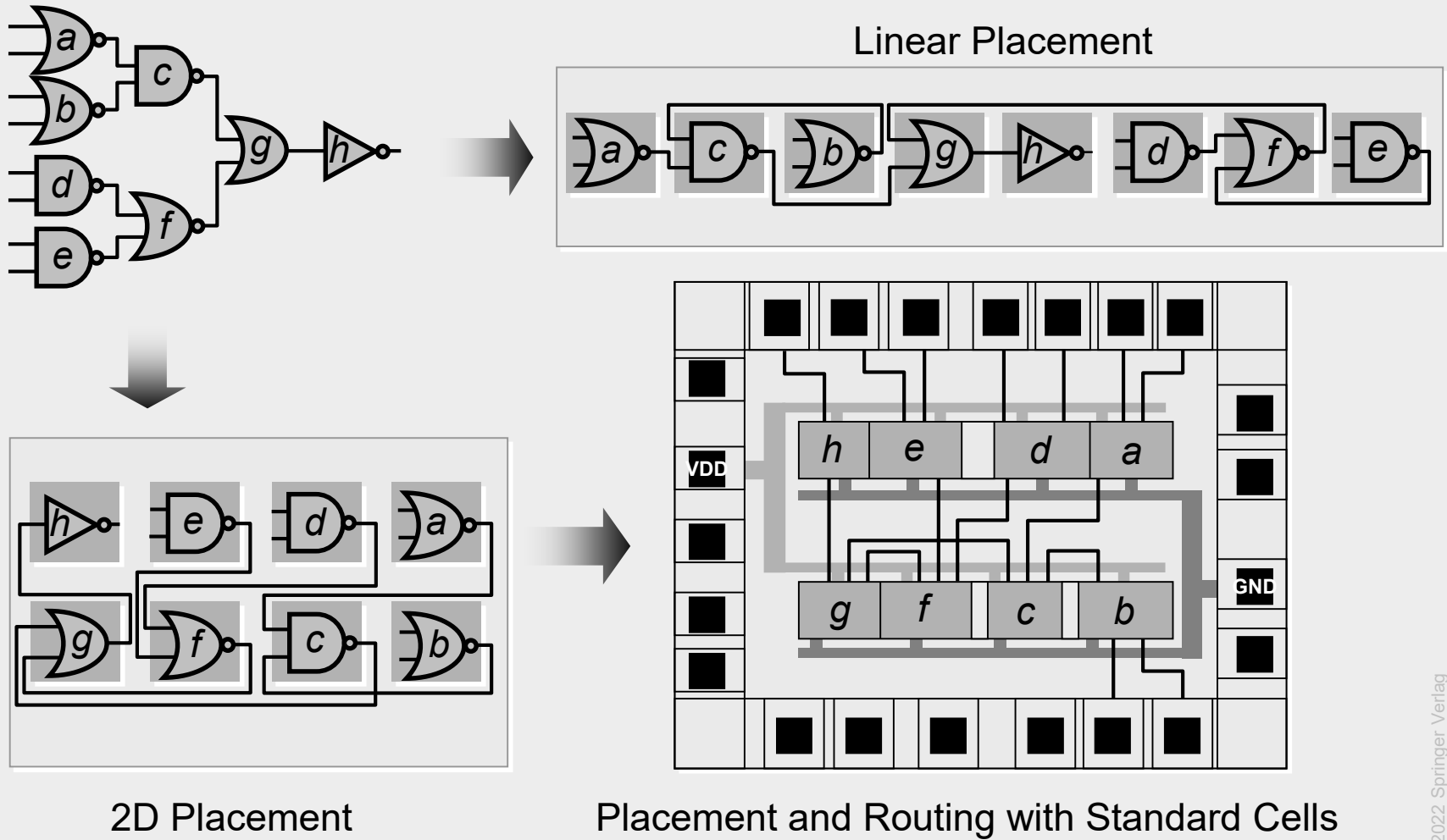
 Springer

- 4.1 Introduction
- 4.2 Optimization Objectives
- 4.3 Global Placement
  - 4.3.1 Min-Cut Placement
  - 4.3.2 Analytic Placement
  - 4.3.3 Simulated Annealing
  - 4.3.4 Modern Placement Algorithms
- 4.4 Legalization and Detailed Placement

# 4.1 Introduction



# 4.1 Introduction

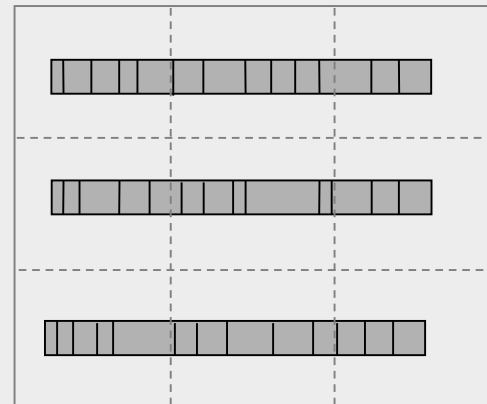


# 4.1 Introduction

Global Placement

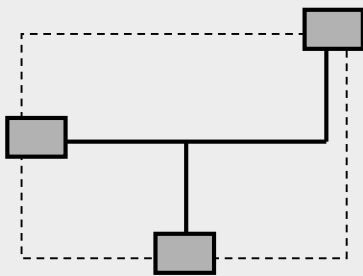


Detailed Placement

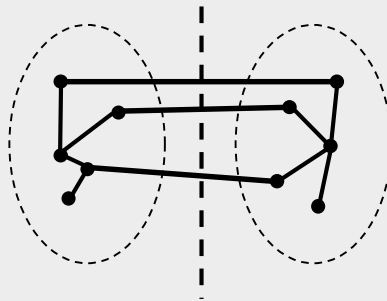


## 4.2 Optimization Objectives

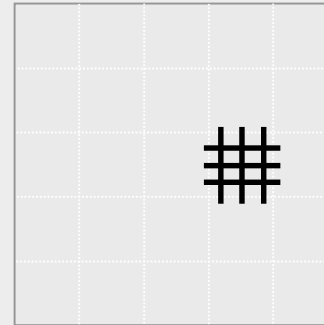
Total  
Wirelength



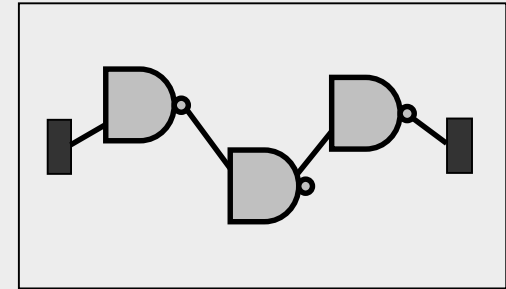
Number of  
Cut Nets



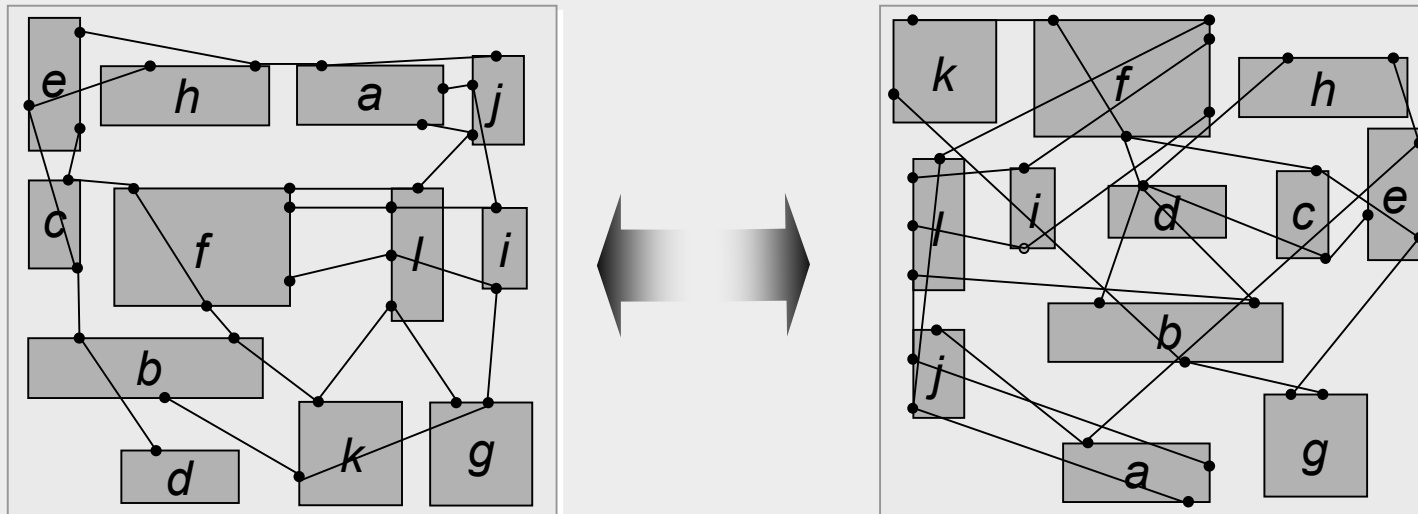
Wire  
Congestion



Signal  
Delay



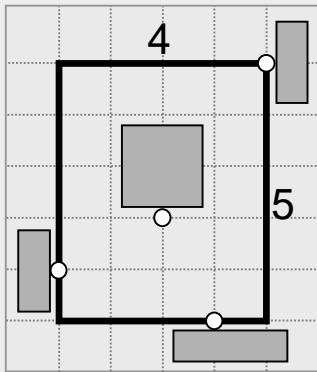
## 4.2 Optimization Objectives – Total Wirelength



## 4.2 Optimization Objectives – Total Wirelength

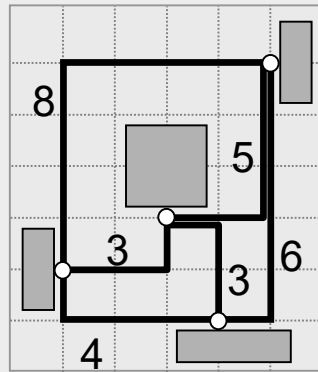
### Wirelength estimation for a given placement

Half-perimeter  
wirelength  
(HPWL)



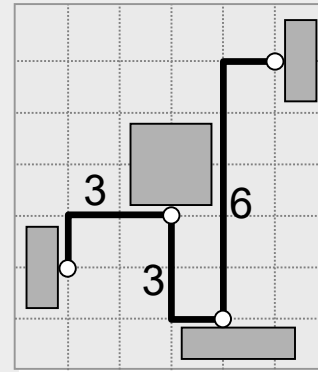
HPWL = 9

Complete  
graph  
(clique)



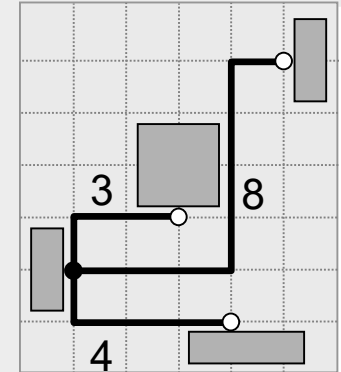
Clique Length =  
 $(2/p)\sum_{e \in \text{clique}} d_M(e) = 14.5$

Monotone  
chain



Chain Length = 12

Star model



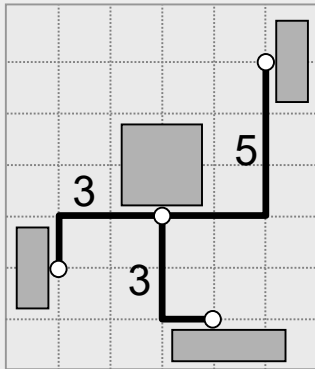
Star Length = 15



## 4.2 Optimization Objectives – Total Wirelength

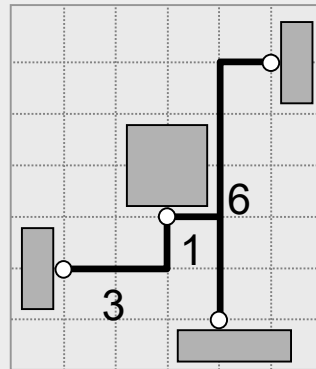
### Wirelength estimation for a given placement (cont'd.)

Rectilinear  
minimum  
spanning  
tree (RMST)



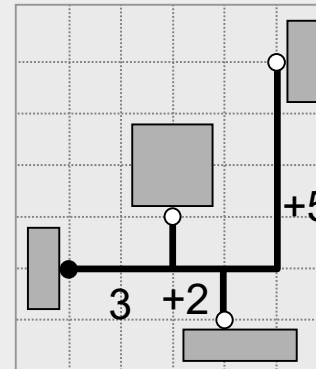
RMST Length = 11

Rectilinear  
Steiner  
minimum  
tree (RSMT)



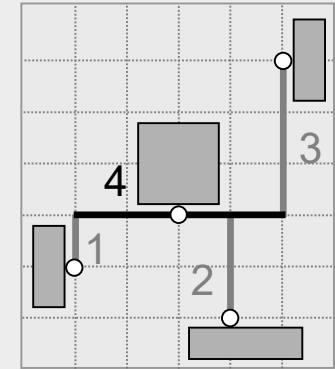
RSMT Length = 10

Rectilinear  
Steiner  
arborescence  
model (RSA)



RSA Length = 10

Single-trunk  
Steiner  
tree (STST)

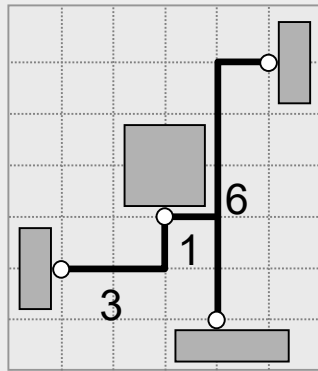


STST Length = 10

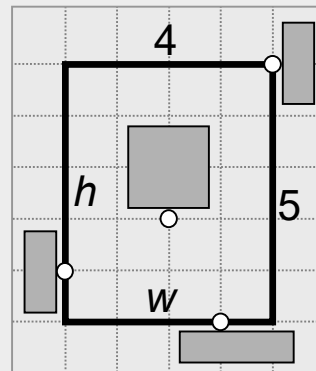
### Wirelength estimation for a given placement (cont'd.)

Preferred method: Half-perimeter wirelength (HPWL)

- Fast (order of magnitude faster than RSMT)
- Equal to length of RSMT for 2- and 3-pin nets
- Margin of error for real circuits approx. 8% [Chu, ICCAD 04]



RSMT Length = 10



HPWL = 9

$$L_{\text{HPWL}} = w + h$$

## 4.2 Optimization Objectives – Total Wirelength

### Total wirelength with net weights (weighted wirelength)

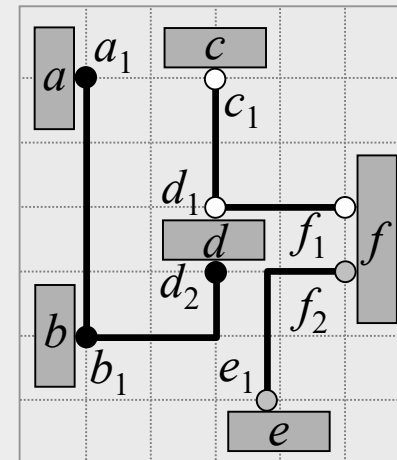
- For a placement  $P$ , an estimate of total weighted wirelength is

$$L(P) = \sum_{net \in P} w(net) \cdot L(net)$$

where  $w(net)$  is the weight of  $net$ , and  $L(net)$  is the estimated wirelength of  $net$ .

- Example:

Nets	Weights
$N_1 = (a_1, b_1, d_2)$	$w(N_1) = 2$
$N_2 = (c_1, d_1, f_1)$	$w(N_2) = 4$
$N_3 = (e_1, f_2)$	$w(N_3) = 1$



$$L(P) = \sum_{net \in P} w(net) \cdot L(net) = 2 \cdot 7 + 4 \cdot 4 + 1 \cdot 3 = 33$$

### Cut sizes of a placement

- To improve total wirelength of a placement  $P$ , separately calculate the number of crossings of global vertical and horizontal cutlines, and minimize

$$L(P) = \sum_{v \in V_P} \Psi_P(v) + \sum_{h \in H_P} \Psi_P(h)$$

where  $\Psi_P(\text{cut})$  be the set of nets cut by a cutline  $\text{cut}$

## 4.2 Optimization Objectives – Number of Cut Nets

### Cut sizes of a placement

- Example:

Nets

$$N_1 = (a_1, b_1, d_2)$$

$$N_2 = (c_1, d_1, f_1)$$

$$N_3 = (e_1, f_2)$$

- Cut values for each global cutline

$$\psi_P(v_1) = 1 \quad \psi_P(v_2) = 2$$

$$\psi_P(h_1) = 3 \quad \psi_P(h_2) = 2$$

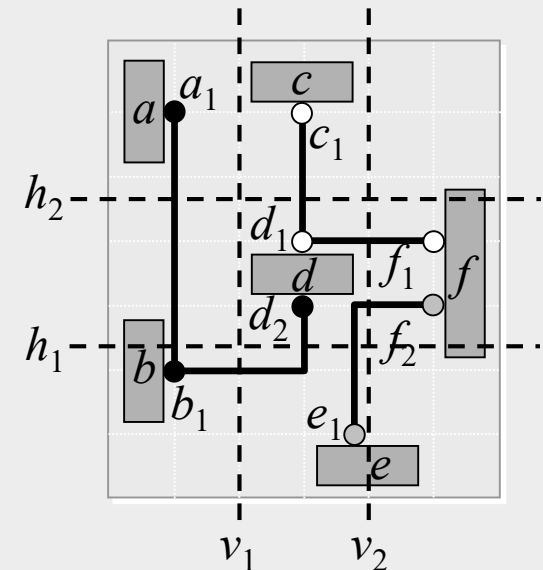
- Total number of crossings in  $P$

$$\psi_P(v_1) + \psi_P(v_2) + \psi_P(h_1) + \psi_P(h_2) = 1 + 2 + 3 + 2 = 8$$

- Cut sizes

$$X(P) = \max(\psi_P(v_1), \psi_P(v_2)) = \max(1, 2) = 2$$

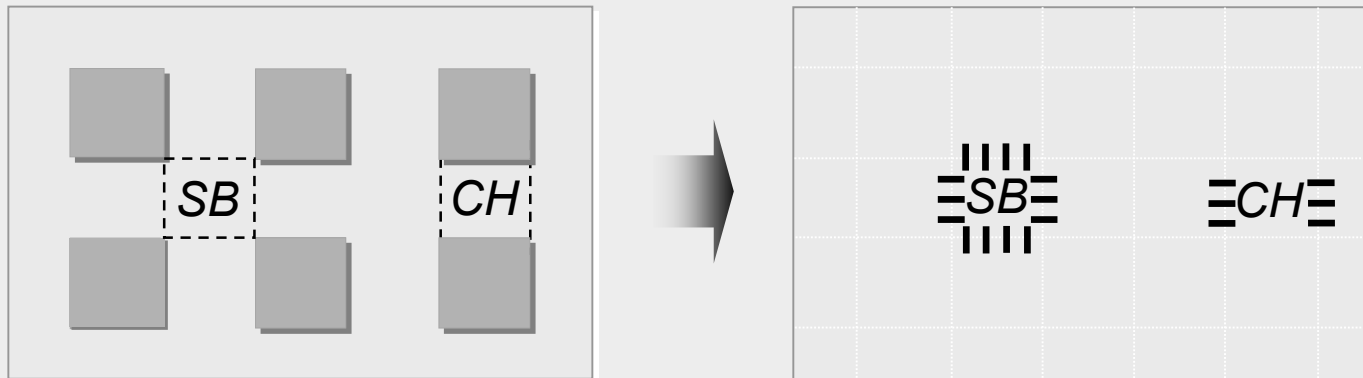
$$Y(P) = \max(\psi_P(h_1), \psi_P(h_2)) = \max(3, 2) = 3$$



## 4.2 Optimization Objectives – Wire Congestion

### Routing congestion of a placement

- Ratio of demand for routing tracks to the supply of available routing tracks
- Estimated by the number of nets that pass through the boundaries of individual routing regions



Wire capacities

## 4.2 Optimization Objectives – Wire Congestion

### Routing congestion of a placement

- Formally, the local wire density  $\varphi_P(e)$  of an edge  $e$  between two neighboring grid cells is

$$\varphi_P(e) = \frac{\eta_P(e)}{\sigma_P(e)}$$

where  $\eta_P(e)$  is the estimated number of nets that cross  $e$  and  $\sigma_P(e)$  is the maximum number of nets that can cross  $e$

- If  $\varphi_P(e) > 1$ , then too many nets are estimated to cross  $e$ , making  $P$  more likely to be unroutable.
- The wire density of  $P$  is  $\Phi(P) = \max_{e \in E}(\varphi_P(e))$

where  $E$  is the set of all edges

- If  $\Phi(P) \leq 1$ , then the design is estimated to be fully routable, otherwise routing will need to detour some nets through less-congested edges

## 4.2 Optimization Objectives – Wire Congestion

### Wire Density of a placement

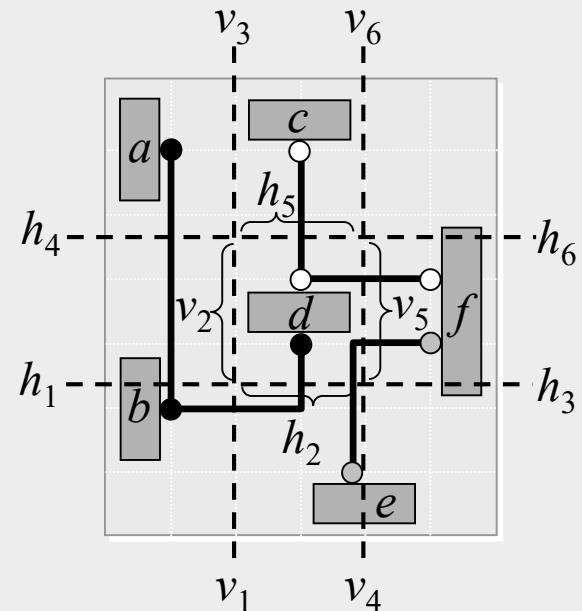
$\eta_P(h_1) = 1$	$\eta_P(v_1) = 1$
$\eta_P(h_2) = 2$	$\eta_P(v_2) = 0$
$\eta_P(h_3) = 0$	$\eta_P(v_3) = 0$
$\eta_P(h_4) = 1$	$\eta_P(v_4) = 0$
$\eta_P(h_5) = 1$	$\eta_P(v_5) = 2$
$\eta_P(h_6) = 0$	$\eta_P(v_6) = 0$

Maximum:  $\eta_P(e) = 2$

$$\Phi(P) = \frac{\eta_P(e)}{\sigma_P(e)} = \frac{2}{3}$$



Routable





### Circuit timing of a placement

- Static timing analysis using actual arrival time ( $AAT$ ) and required arrival time ( $RAT$ )
  - $AAT(v)$  represents the latest transition time at a given node  $v$  measured from the beginning of the clock cycle
  - $RAT(v)$  represents the time by which the latest transition at  $v$  must complete in order for the circuit to operate correctly within a given clock cycle.
- For correct operation of the chip with respect to setup (maximum path delay) constraints, it is required that  $AAT(v) \leq RAT(v)$ .

4.1 Introduction

4.2 Optimization Objectives

 4.3 Global Placement

- 4.3.1 Min-Cut Placement
- 4.3.2 Analytic Placement
- 4.3.3 Simulated Annealing
- 4.3.4 Modern Placement Algorithms

4.4 Legalization and Detailed Placement

- **Partitioning-based algorithms:**
  - The netlist and the layout are divided into smaller sub-netlists and sub-regions, respectively
  - Process is repeated until each sub-netlist and sub-region is small enough to be handled optimally
  - Detailed placement often performed by optimal solvers, facilitating a natural transition from global placement to detailed placement
  - Example: min-cut placement
- **Analytic techniques:**
  - Model the placement problem using an objective (cost) function, which can be optimized via numerical analysis
  - Examples: quadratic placement and force-directed placement
- **Stochastic algorithms:**
  - Randomized moves that allow hill-climbing are used to optimize the cost function
  - Example: simulated annealing

# Global Placement

Partitioning-based

Analytic

Stochastic

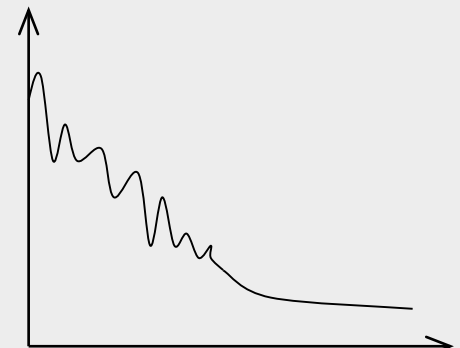
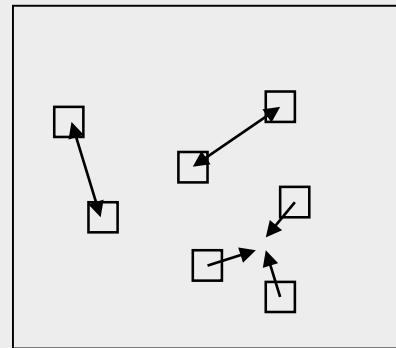
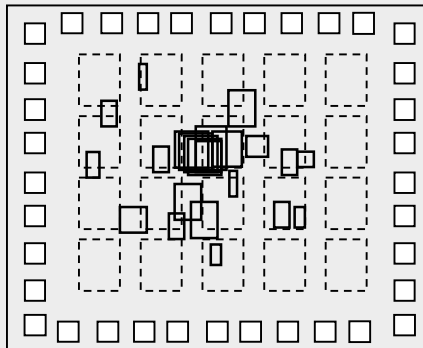
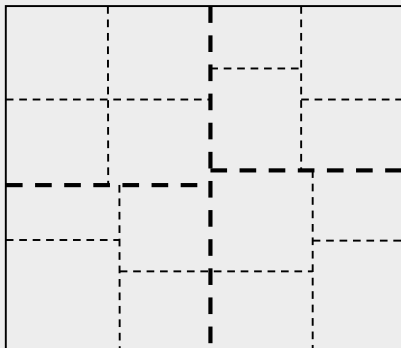


Min-cut  
placement

Quadratic  
placement

Force-directed  
placement

Simulated annealing

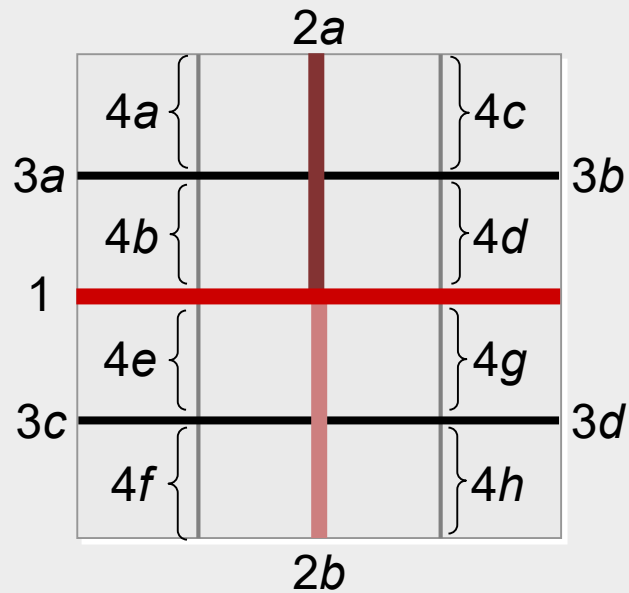


## 4.3.1 Min-Cut Placement

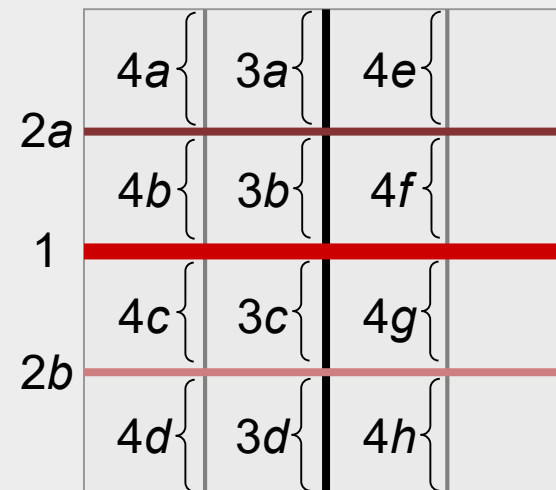
- Uses partitioning algorithms to divide (1) the netlist and (2) the layout region into smaller sub-netlists and sub-regions
- Conceptually, each sub-region is assigned a portion of the original netlist
- Each cut heuristically minimizes the number of cut nets using, for example,
  - Kernighan-Lin (KL) algorithm
  - Fiduccia-Mattheyses (FM) algorithm

## 4.3.1 Min-Cut Placement

Alternating cutline directions



Repeating cutline directions



## 4.3.1 Min-Cut Placement

**Input:** netlist *Netlist*, layout area *LA*, minimum number of cells per region *cells\_min*

**Output:** placement *P*

*P* =  $\emptyset$

*regions* = ASSIGN(*Netlist*,*LA*)

**while** (*regions*  $\neq$   $\emptyset$ )

*region* = FIRST\_ELEMENT(*regions*)

    REMOVE(*regions*, *region*)

**if** (*region* contains more than *cell\_min* cells)

        (*sr1*,*sr2*) = BISECT(*region*)

        ADD\_TO\_END(*regions*,*sr1*)

        ADD\_TO\_END(*regions*,*sr2*)

**else**

        PLACE(*region*)

        ADD(*P*,*region*)

// assign netlist to layout area

// while regions still not placed

// first element in *regions*

// remove first element of *regions*

// divide *region* into two subregions

// *sr1* and *sr2*, obtaining the sub-

// netlists and sub-areas

// add *sr1* to the end of *regions*

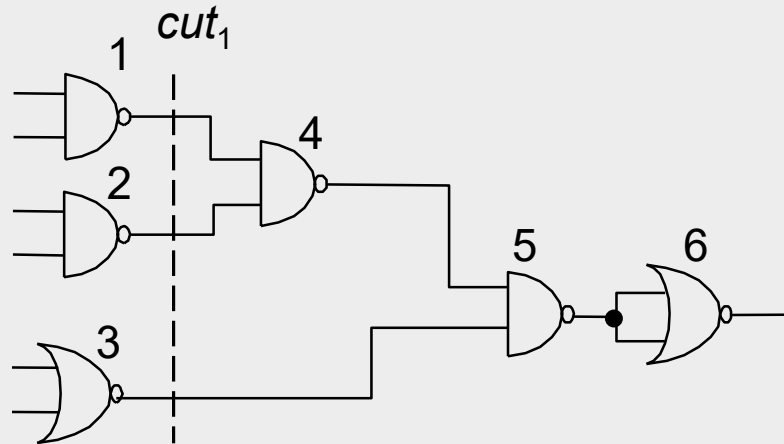
// add *sr2* to the end of *regions*

// place *region*

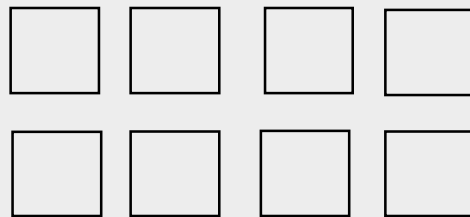
// add *region* to *P*

## 4.3.1 Min-Cut Placement – Example

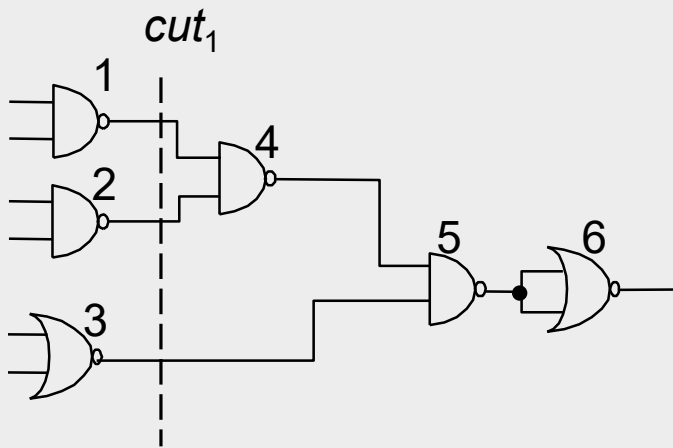
Given:



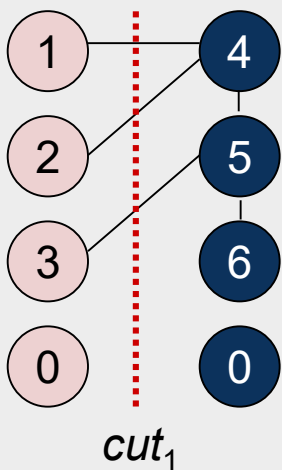
Task: 4 x 2 placement with minimum wirelength using alternative cutline directions and the KL algorithm



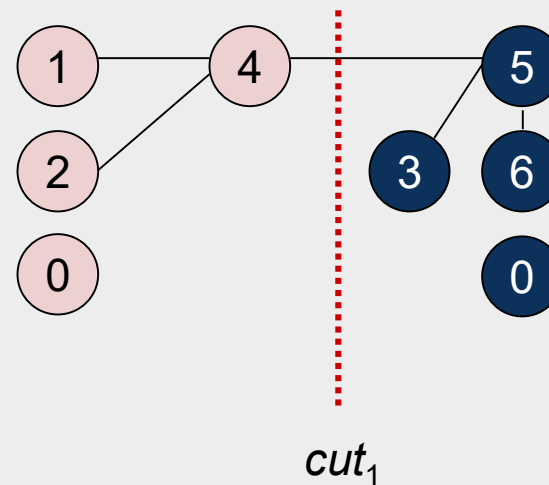


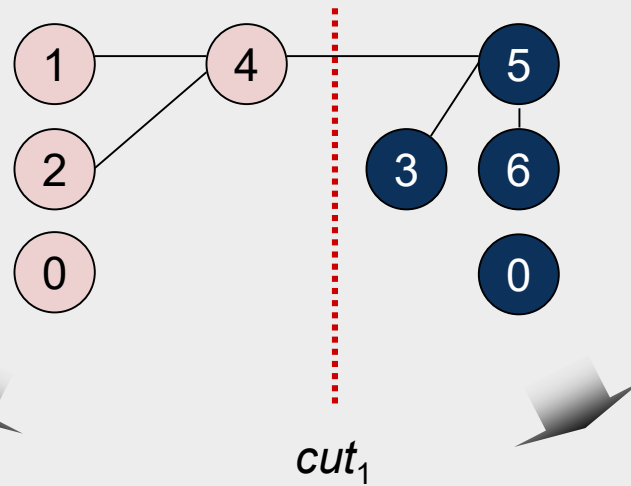


Vertical cut  $cut_1$ :  $L=\{1,2,3\}$ ,  $R=\{4,5,6\}$



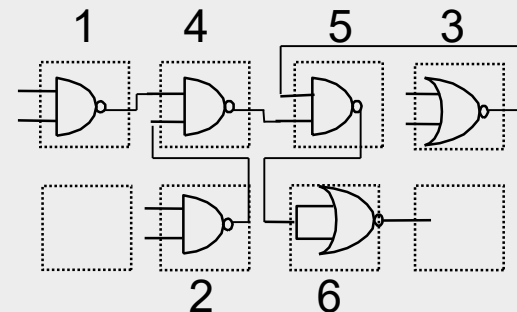
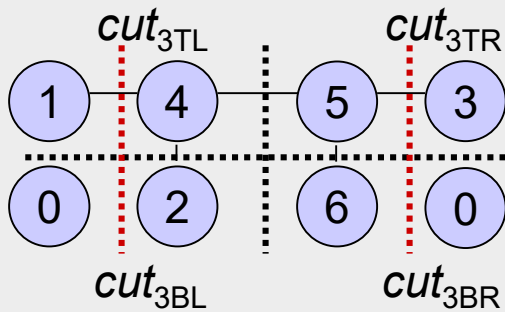
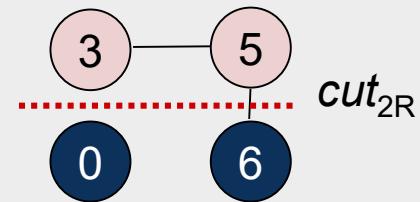
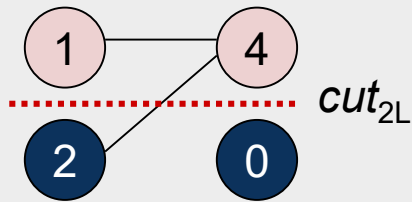
KL Algorithm



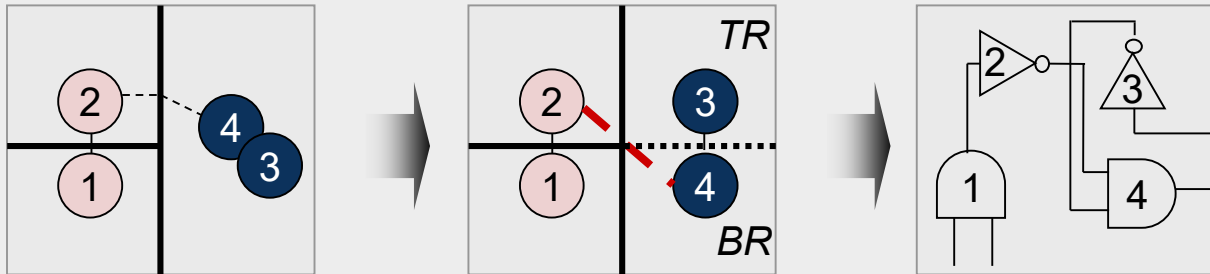


Horizontal cut  $cut_{2L}$ :  $T=\{1,4\}$ ,  $B=\{2,0\}$

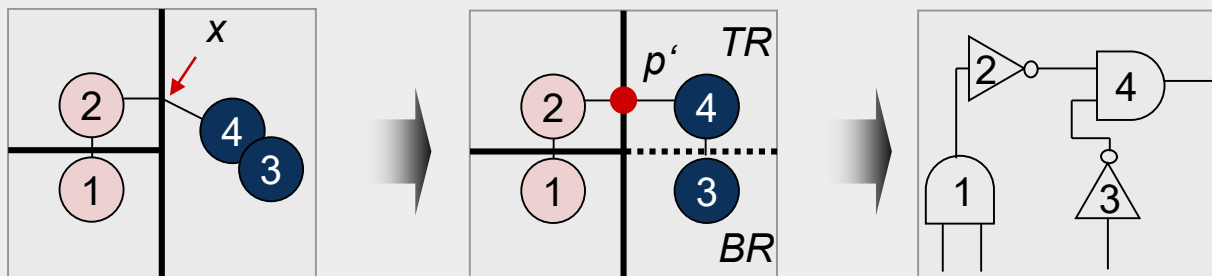
Horizontal cut  $cut_{2R}$ :  $T=\{3,5\}$ ,  $B=\{6,0\}$



## 4.3.1 Min-Cut Placement – Terminal Propagation



- Terminal Propagation
  - External connections are represented by artificial connection points on the cutline
  - Dummy nodes in hypergraphs



## 4.3.1 Min-Cut Placement

- Advantages:
  - Reasonably fast
  - Objective function can be adjusted, e.g., to perform timing-driven placement
  - Hierarchical strategy applicable to large circuits
- Disadvantages:
  - Randomized, chaotic algorithms – small changes in input lead to large changes in output
  - Optimizing one cutline at a time may result in routing congestion elsewhere

## 4.3.2 Analytic Placement – Quadratic Placement

- Objective function is quadratic; sum of (weighted) **squared Euclidean distance** represents placement objective function

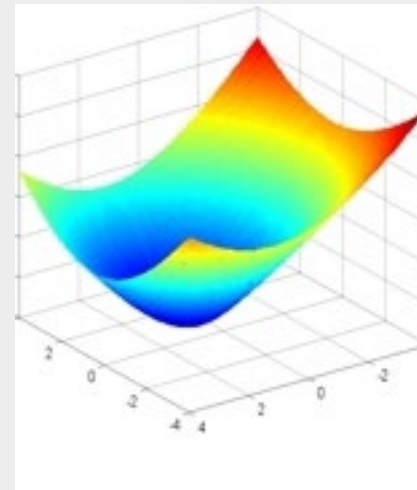
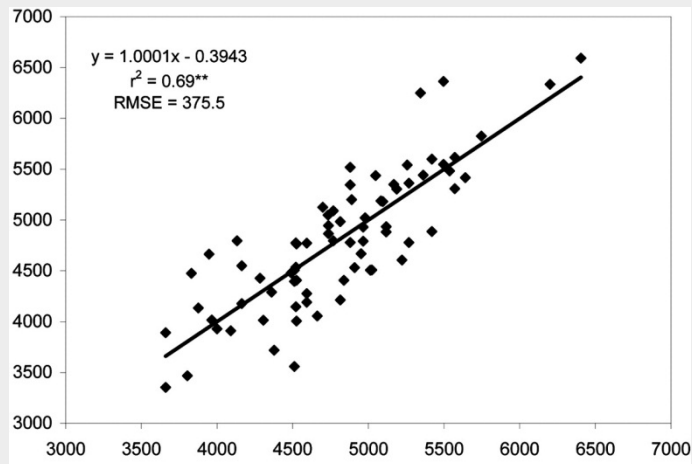
$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left( (x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where  $n$  is the total number of cells, and  $c(i,j)$  is the connection cost between cells  $i$  and  $j$ .

- Only two-point-connections
- Minimize objective function by equating its derivative to zero which reduces to solving a system of linear equations

## 4.3.2 Analytic Placement – Quadratic Placement

- Similar to Least-Mean-Square Method (root mean square)
- Build error function with analytic form:  $E(a,b) = \sum (a \cdot x_i + b - y_i)^2$



## 4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left( (x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where  $n$  is the total number of cells, and  $c(i,j)$  is the connection cost between cells  $i$  and  $j$ .

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1, j=1}^n c(i, j)(x_i - x_j)^2 \quad L_y(P) = \sum_{i=1, j=1}^n c(i, j)(y_i - y_j)^2$$

- Convex quadratic optimization problem: any local minimum solution is also a global minimum
- Optimal  $x$ - and  $y$ -coordinates can be found by setting the partial derivatives of  $L_x(P)$  and  $L_y(P)$  to zero

## 4.3.2 Analytic Placement – Quadratic Placement


$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left( (x_i - x_j)^2 + (y_i - y_j)^2 \right)$$


where  $n$  is the total number of cells, and  $c(i,j)$  is the connection cost between cells  $i$  and  $j$ .

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1,j=1}^n c(i,j)(x_i - x_j)^2$$

$$L_y(P) = \sum_{i=1,j=1}^n c(i,j)(y_i - y_j)^2$$


$$\frac{\partial L_x(P)}{\partial X} = AX - b_x = 0$$


$$\frac{\partial L_y(P)}{\partial Y} = AY - b_y = 0$$

where  $A$  is a matrix with  $A[i][j] = -c(i,j)$  when  $i \neq j$ ,  
and  $A[i][i] =$  the sum of incident connection weights of cell  $i$ .

$X$  is a vector of all the  $x$ -coordinates of the non-fixed cells, and  $b_x$  is a vector with  $b_x[i] =$  the sum of  $x$ -coordinates of all fixed cells attached to  $i$ .

$Y$  is a vector of all the  $y$ -coordinates of the non-fixed cells, and  $b_y$  is a vector with  $b_y[i] =$  the sum of  $y$ -coordinates of all fixed cells attached to  $i$ .



## 4.3.2 Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2} \sum_{i,j=1}^n c_{ij} \left( (x_i - x_j)^2 + (y_i - y_j)^2 \right)$$

where  $n$  is the total number of cells, and  $c(i,j)$  is the connection cost between cells  $i$  and  $j$ .

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i,j=1}^n c(i,j)(x_i - x_j)^2$$



$$\frac{\partial L_x(P)}{\partial X} = AX - b_x = 0$$

$$L_y(P) = \sum_{i,j=1}^n c(i,j)(y_i - y_j)^2$$

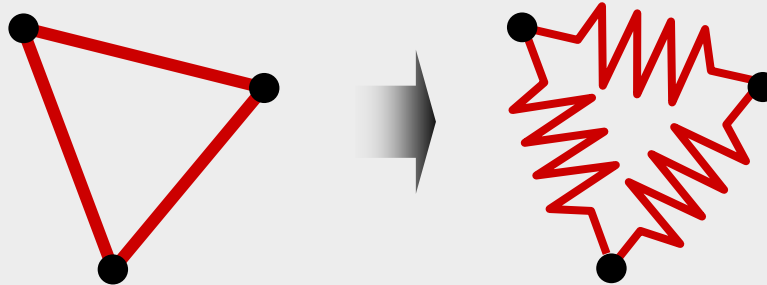


$$\frac{\partial L_y(P)}{\partial Y} = AY - b_y = 0$$

- System of linear equations for which iterative numerical methods can be used to find a solution

## 4.3.2 Analytic Placement – Quadratic Placement

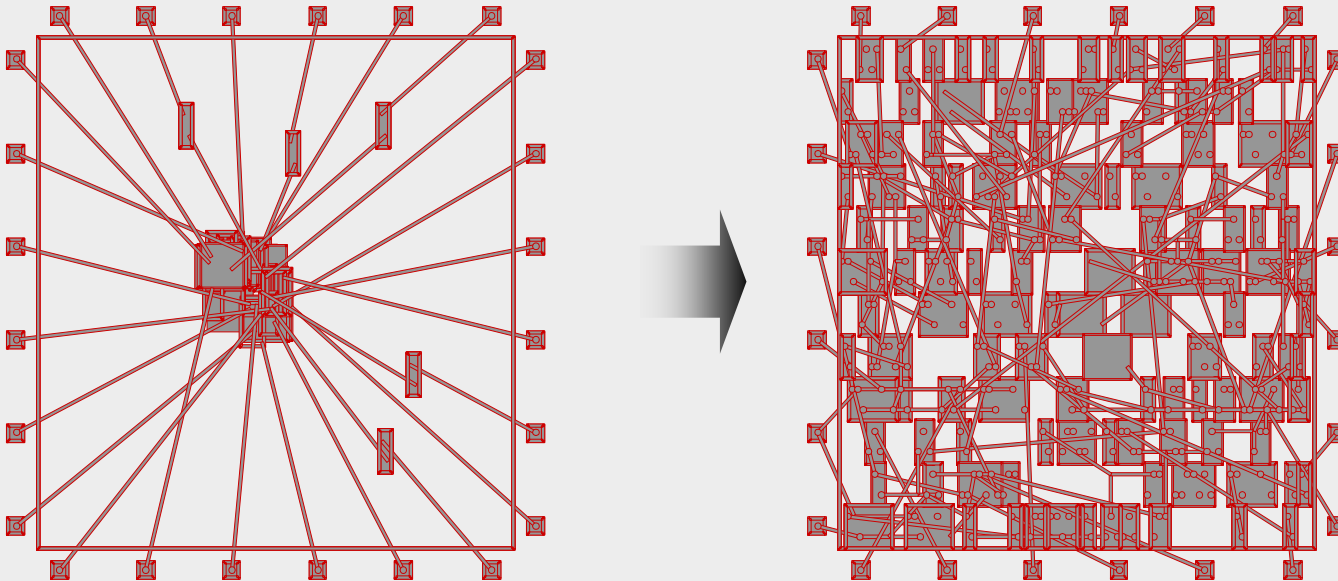
- Mechanical analogy: mass-spring system



- Squared Euclidean distance is proportional to the energy of a spring between these points
  - Quadratic objective function represents total energy of the spring system; for each movable object, the  $x$  ( $y$ ) partial derivative represents the total force acting on that object
  - Setting the forces of the nets to zero, an equilibrium state is mathematically modeled that is characterized by zero forces acting on each movable object
  - At the end, all springs are in a force equilibrium with a minimal total spring energy; this equilibrium represents the minimal sum of squared wirelength
- Result: many cell overlaps

## 4.3.2 Analytic Placement – Quadratic Placement

- Second stage of quadratic placers: cells are spread out to remove overlaps
- Methods:
  - Adding fake nets that pull cells away from dense regions toward anchors
  - Geometric sorting and scaling
  - Repulsion forces, etc.

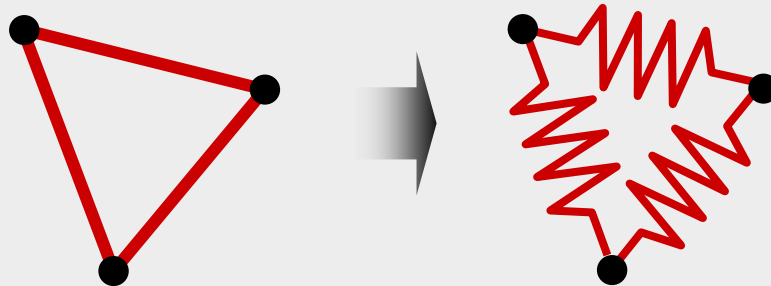


## 4.3.2 Analytic Placement – Quadratic Placement

- Advantages:
  - Captures the placement problem concisely in mathematical terms
  - Leverages efficient algorithms from numerical analysis and available software
  - Can be applied to large circuits without netlist clustering (flat)
  - Stability: small changes in the input do not lead to large changes in the output
- Disadvantages:
  - Connections to fixed objects are necessary: I/O pads, pins of fixed macros, etc.

## 4.3.2 Analytic Placement – Force-directed Placement

- Cells and wires are modeled using the mechanical analogy of a mass-spring system, i.e., masses connected to Hooke's-Law springs



- Attraction force between cells is directly proportional to their distance
- Cells will eventually settle in a **force equilibrium** → minimized wirelength

## 4.3.2 Analytic Placement – Force-directed Placement

- Given two connected cells  $a$  and  $b$ , the attraction force  $\vec{F}_{ab}$  exerted on  $a$  by  $b$  is

$$\vec{F}_{ab} = c(a, b) \cdot (\vec{b} - \vec{a})$$

where

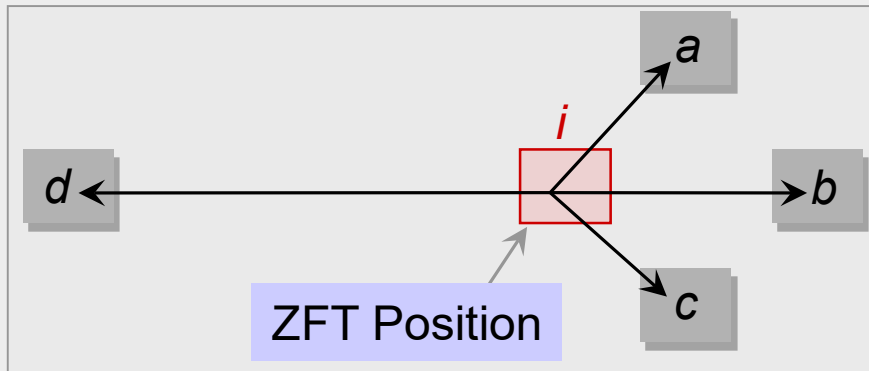
- $c(a, b)$  is the connection weight (priority) between cells  $a$  and  $b$ , and
  - $(\vec{b} - \vec{a})$  is the vector difference of the positions of  $a$  and  $b$  in the Euclidean plane
- The sum of forces exerted on a cell  $i$  connected to other cells  $1 \dots j$  is

$$\vec{F}_i = \sum_{c(i, j) \neq 0} \vec{F}_{ij}$$

- Zero-force target (ZFT):** position that minimizes this sum of forces

## 4.3.2 Analytic Placement – Force-directed Placement

Zero-Force-Target (ZFT) position of cell  $i$



$$\min \vec{F}_i = c(i,a) \cdot (\vec{a} - \vec{i}) + c(i,b) \cdot (\vec{b} - \vec{i}) + c(i,c) \cdot (\vec{c} - \vec{i}) + c(i,d) \cdot (\vec{d} - \vec{i})$$

## 4.3.2 Analytic Placement – Force-directed Placement

### Basic force-directed placement

- Iteratively moves all cells to their respective ZFT positions
- $x$ - and  $y$ -direction forces are set to zero:

$$\sum_{c(i,j) \neq 0} c(i,j) \cdot (x_j^0 - x_i^0) = 0 \quad \sum_{c(i,j) \neq 0} c(i,j) \cdot (y_j^0 - y_i^0) = 0$$

- Rearranging the variables to solve for  $x_i^0$  and  $y_i^0$  yields

$$x_i^0 = \frac{\sum_{c(i,j) \neq 0} c(i,j) \cdot x_j^0}{\sum_{c(i,j) \neq 0} c(i,j)}$$

$$y_i^0 = \frac{\sum_{c(i,j) \neq 0} c(i,j) \cdot y_j^0}{\sum_{c(i,j) \neq 0} c(i,j)}$$

Computation of  
ZFT position of cell  $i$   
connected with  
cells 1 ...  $j$



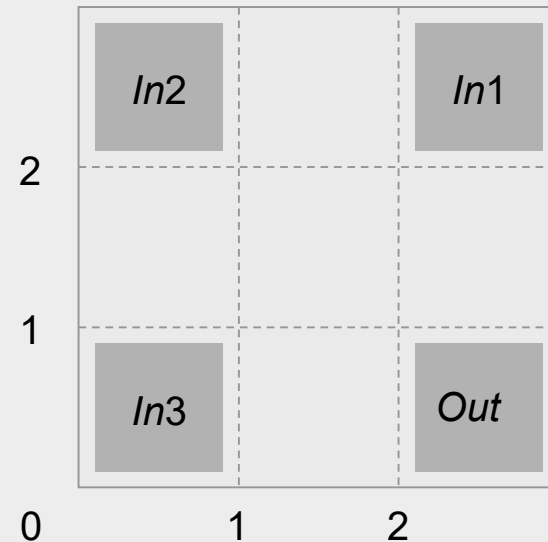
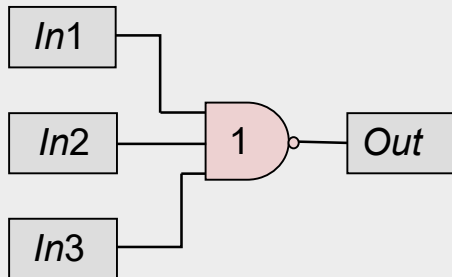
## 4.3.2 Analytic Placement – Force-directed Placement

### Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3 x 3 grid
- Pad positions:  $In1$  (2,2),  $In2$  (0,2),  $In3$  (0,0),  $Out$  (2,0)
- Weighted connections:  $c(a,In1) = 8$ ,  $c(a,In2) = 10$ ,  $c(a,In3) = 2$ ,  $c(a,Out) = 2$

Task: find the ZFT position of cell  $a$



## 4.3.2 Analytic Placement – Force-directed Placement

### Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3 x 3 grid
- Pad positions:  $In1$  (2,2),  $In2$  (0,2),  $In3$  (0,0),  $Out$  (2,0)

Solution:

$$x_a^0 = \frac{\sum_{c(i,j) \neq 0} c(a,j) \cdot x_j^0}{\sum_{c(i,j) \neq 0} c(a,j)} = \frac{c(a, In1) \cdot x_{In1} + c(a, In2) \cdot x_{In2} + c(a, In3) \cdot x_{In3} + c(a, Out) \cdot x_{Out}}{c(a, In1) + c(a, In2) + c(a, In3) + c(a, Out)} = \frac{8 \cdot 2 + 10 \cdot 0 + 2 \cdot 0 + 2 \cdot 2}{8 + 10 + 2 + 2} = \frac{20}{22} \approx 0.9$$

$$y_a^0 = \frac{\sum_{c(i,j) \neq 0} c(a,j) \cdot y_j^0}{\sum_{c(i,j) \neq 0} c(a,j)} = \frac{c(a, In1) \cdot y_{In1} + c(a, In2) \cdot y_{In2} + c(a, In3) \cdot y_{In3} + c(a, Out) \cdot y_{Out}}{c(a, In1) + c(a, In2) + c(a, In3) + c(a, Out)} = \frac{8 \cdot 2 + 10 \cdot 2 + 2 \cdot 0 + 2 \cdot 0}{8 + 10 + 2 + 2} = \frac{36}{22} \approx 1.6$$

ZFT position of cell  $a$  is (1,2)

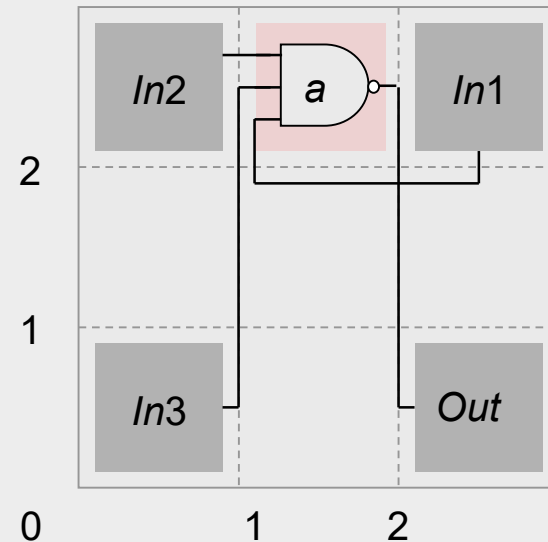
## 4.3.2 Analytic Placement – Force-directed Placement

### Example: ZFT position

Given:

- Circuit with NAND gate 1 and four I/O pads on a 3 x 3 grid
- Pad positions:  $In1$  (2,2),  $In2$  (0,2),  $In3$  (0,0),  $Out$  (2,0)

Solution:



ZFT position of cell  $a$  is (1,2)

## 4.3.2 Analytic Placement – Force-directed Placement

**Input:** set of all cells  $V$

**Output:** placement  $P$

```
P = PLACE(V)
loc = LOCATIONS(P)
foreach (cell c ∈ V)
    status[c] = UNMOVED
while (ALL_MOVED(V) || !STOP())

    c = MAX_DEGREE(V,status)

    ZFT_pos = ZFT_POSITION(c)
    if (loc[ZFT_pos] == ∅)
        loc[ZFT_pos] = c
    else
        RELOCATE(c,loc)
    status[c] = MOVED
```

// arbitrary initial placement  
// set coordinates for each cell in  $P$   
  
// continue until all cells have been  
// moved or some stopping  
// criterion is reached  
// unmoved cell that has largest  
// number of connections  
// ZFT position of  $c$   
// if position is unoccupied,  
// move  $c$  to its ZFT position  
  
// use methods discussed next  
// mark  $c$  as moved

## 4.3.2 Analytic Placement – Force-directed Placement

Finding a valid location for a cell with an occupied ZFT position  
( $p$ : incoming cell,  $q$ : cell in  $p$ 's ZFT position)

- If possible, move  $p$  to a cell position close to  $q$ .
- Chain move: cell  $p$  is moved to cells  $q$ 's location.
  - Cell  $q$ , in turn, is shifted to the next position. If a cell  $r$  is occupying this space, cell  $r$  is shifted to the next position.
  - This continues until all affected cells are placed.
- Compute the cost difference if  $p$  and  $q$  were to be swapped.  
If the total cost reduces, i.e., the weighted connection length  $L(P)$  is smaller, then swap  $p$  and  $q$ .

## 4.3.2 Analytic Placement – Force-directed Placement (Example)

Given:

Nets

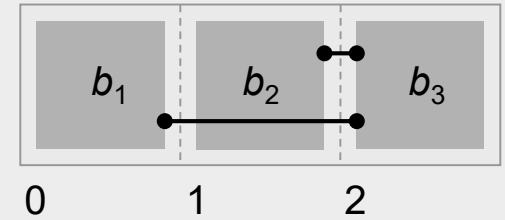
$N_1 = (b_1, b_3)$

$N_2 = (b_2, b_3)$

Weight

$c(N_1) = 2$

$c(N_2) = 1$



## 4.3.2 Analytic Placement – Force-directed Placement (Example)

Given:

Nets

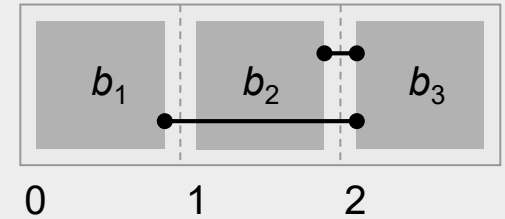
$$N_1 = (b_1, b_3)$$

$$N_2 = (b_2, b_3)$$

Weight

$$c(N_1) = 2$$

$$c(N_2) = 1$$



Incoming cell  $p$

ZFT position of cell  $p$

Cell  $q$

$L(P)$  before move

$L(P)$  / placement after move

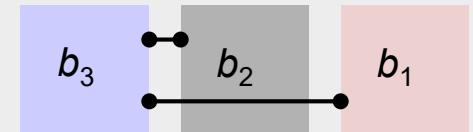
$b_3$

$$x_{b_3}^0 = \frac{\sum_{c(b_3,j) \neq 0} c(b_3,j) \cdot x_j^0}{\sum_{c(b_3,j) \neq 0} c(b_3,j)} = \frac{2 \cdot 0 + 1 \cdot 1}{2 + 1} \approx 0$$

$b_1$

$$L(P) = 5$$

$$L(P) = 5$$

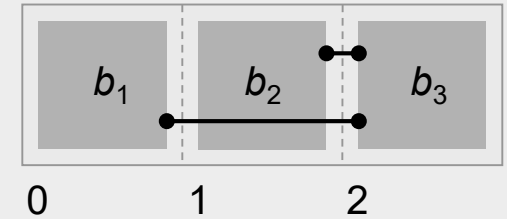


$\Rightarrow$  No swapping of  $b_3$  and  $b_1$

## 4.3.2 Analytic Placement – Force-directed Placement (Example)

Given:

Nets  
 $N_1 = (b_1, b_3)$       Weight  $c(N_1) = 2$   
 $N_2 = (b_2, b_3)$       Weight  $c(N_2) = 1$



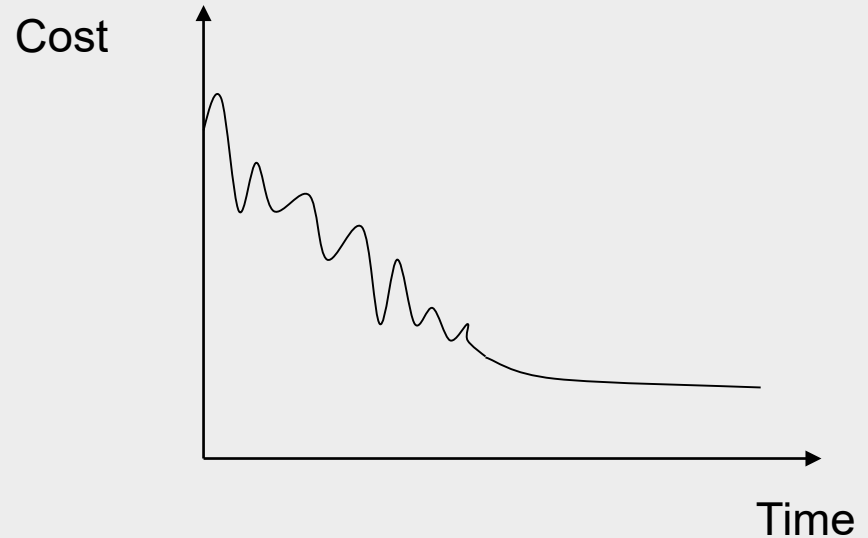
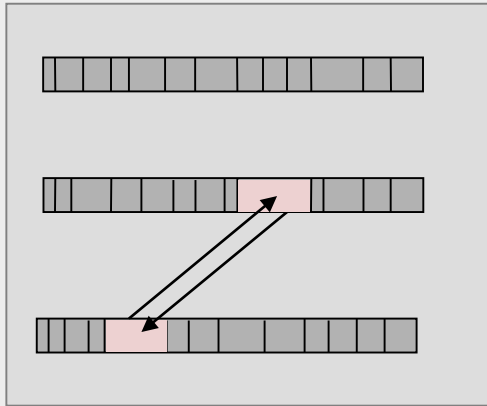
Incoming cell $p$	ZFT position of cell $p$	Cell $q$	$L(P)$ before move	$L(P)$ / placement after move
$b_3$	$x_{b_3}^0 = \frac{\sum_{c(b_3,j) \neq 0} c(b_3,j) \cdot x_j^0}{\sum_{c(b_3,j) \neq 0} c(b_3,j)} = \frac{2 \cdot 0 + 1 \cdot 1}{2 + 1} \approx 0$	$b_1$	$L(P) = 5$	$L(P) = 5$ → No swapping of $b_3$ and $b_1$
$b_2$	$x_{b_2}^0 = \frac{\sum_{c(b_2,j) \neq 0} c(b_2,j) \cdot x_j^0}{\sum_{c(b_2,j) \neq 0} c(b_2,j)} = \frac{1 \cdot 2}{1} = 2$	$b_3$	$L(P) = 5$	$L(P) = 3$ → Swapping of $b_2$ and $b_3$



## 4.3.2 Analytic Placement – Force-directed Placement

- Advantages:
  - Conceptually simple, easy to implement
  - Primarily intended for global placement, but can also be adapted to detailed placement
- Disadvantages:
  - Does not scale to large placement instances
  - Is not very effective in spreading cells in densest regions
  - Poor trade-off between solution quality and runtime
- In practice, FDP is extended by specialized techniques for cell spreading
  - This facilitates scalability and makes FDP competitive

### 4.3.3 Simulated Annealing



- Analogous to the physical **annealing process**
  - Melt metal and then slowly cool it
  - Result: energy-minimal crystal structure
- Modification of an initial configuration (placement) by moving/exchanging of randomly selected cells
  - Accept the new placement if it improves the objective function
  - If no improvement: Move/exchange is accepted with temperature-dependent (i.e., decreasing) probability

### 4.3.3 Simulated Annealing – Algorithm

**Input:** set of all cells  $V$

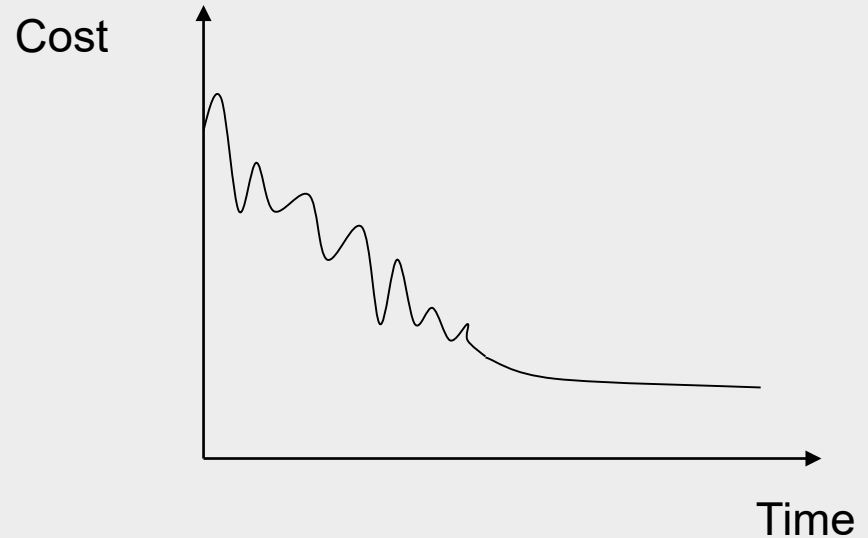
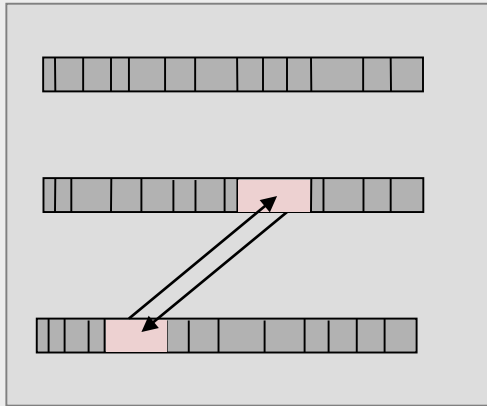
**Output:** placement  $P$

```
 $T = T_0$  // set initial temperature
 $P = \text{PLACE}(V)$  // arbitrary initial placement
while ( $T > T_{min}$ ) // not yet in equilibrium at  $T$ 
  while (!STOP())
     $new\_P = \text{PERTURB}(P)$ 
     $\Delta cost = \text{COST}(new\_P) - \text{COST}(P)$ 
    if ( $\Delta cost < 0$ ) // cost improvement
       $P = new\_P$  // accept new placement
    else // no cost improvement
       $r = \text{RANDOM}(0,1)$  // random number [0,1)
      if ( $r < e^{-\Delta cost/T}$ ) // probabilistically accept
         $P = new\_P$ 
   $T = \alpha \cdot T$  // reduce  $T$ ,  $0 < \alpha < 1$ 
```

### 4.3.3 Simulated Annealing

- Advantages:
  - Can find global optimum (given sufficient time)
  - Well-suited for detailed placement
- Disadvantages:
  - Very slow
  - To achieve high-quality implementation, laborious parameter tuning is necessary
  - Randomized, chaotic algorithms - small changes in the input lead to large changes in the output
- Practical applications of SA:
  - Very small placement instances with complicated constraints
  - Detailed placement, where SA can be applied in small windows (not common anymore)
  - FPGA layout, where complicated constraints are becoming a norm

### 4.3.3 Simulated Annealing



- Analogous to the physical **annealing process**
  - Melt metal and then slowly cool it
  - Result: energy-minimal crystal structure
- Modification of an initial configuration (placement) by moving/exchanging of randomly selected cells
  - Accept the new placement if it improves the objective function
  - If no improvement: Move/exchange is accepted with temperature-dependent (i.e., decreasing) probability

## 4.3.4 Modern Placement Algorithms

- Predominantly analytic algorithms
- Solve two challenges: interconnect minimization and cell overlap removal (spreading)
- Two families:



Quadratic placers



Non-convex  
optimization placers

## 4.3.4 Modern Placement Algorithms



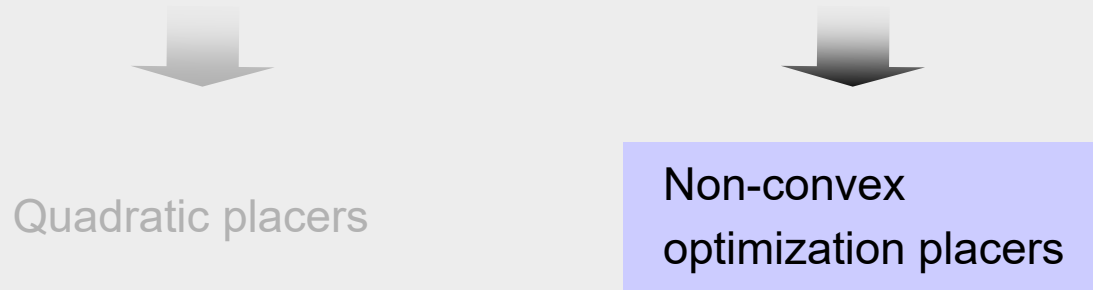
Quadratic placers



Non-convex  
optimization placers

- Solve large, sparse systems of linear equations (formulated using force-directed placement) by the Conjugate Gradient algorithm
- Perform cell spreading by adding fake nets that pull cells away from dense regions toward carefully placed anchors

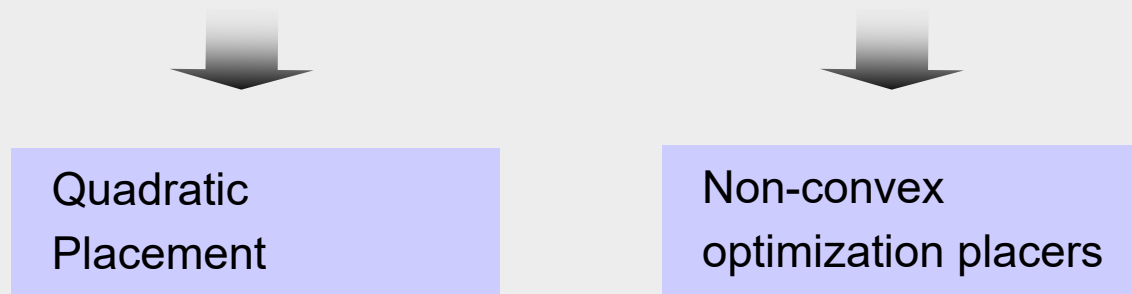
## 4.3.4 Modern Placement Algorithms



- Model interconnect by sophisticated differentiable functions, e.g., log-sum-exp is the popular choice
- Model cell overlap and fixed obstacles by additional (non-convex) functional terms
- Optimize interconnect by the non-linear Conjugate Gradient algorithm
- Sophisticated, slow algorithms
- All leading placers in this category use netlist clustering to improve computational scalability (this further complicates the implementation)



## 4.3.4 Modern Placement Algorithms



Pros and cons:

- Quadratic placers are simpler and faster, easier to parallelize
- Non-convex optimizers tend to produce better solutions
- As of 2011, quadratic placers are catching up in solution quality while running 5-6 times faster <sup>[1]</sup>

## 4.4 Legalization and Detailed Placement

4.1 Introduction

4.2 Optimization Objectives


4.3 Global Placement

4.3.1 Min-Cut Placement

4.3.2 Analytic Placement

4.3.3 Simulated Annealing

4.3.4 Modern Placement Algorithms

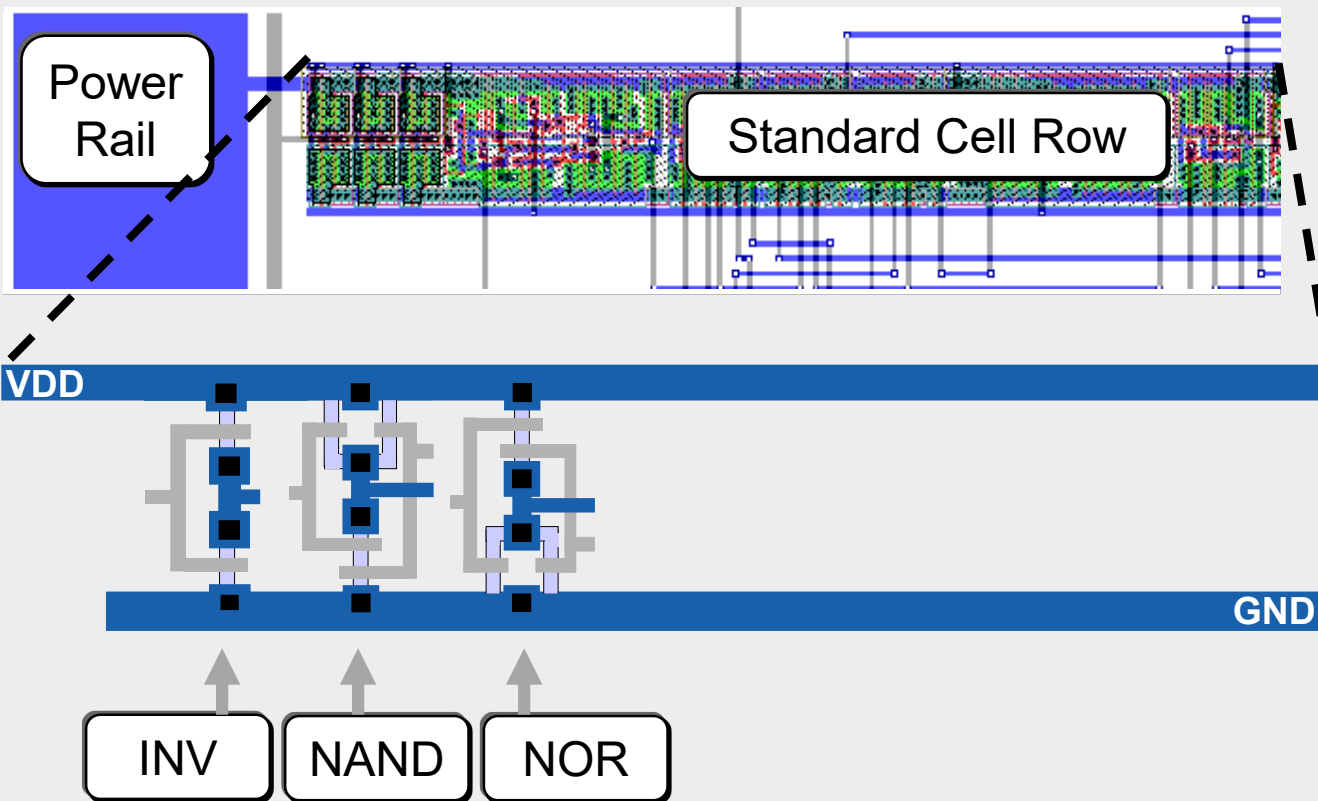
 4.4 Legalization and Detailed Placement

## 4.4 Legalization and Detailed Placement

- Global placement must be legalized
  - Cell locations typically do not align with power rails
  - Small cell overlaps due to incremental changes, such as cell resizing or buffer insertion
- **Legalization** seeks to find legal, non-overlapping placements for all placeable modules
- Legalization can be improved by **detailed placement** techniques, such as
  - Swapping neighboring cells to reduce wirelength
  - Sliding cells to unused space
- Software implementations of legalization and detailed placement are often bundled

## 4.4 Legalization and Detailed Placement

Legal positions of standard cells between VDD and GND rails



# Summary of Chapter 4 – Problem Formulation and Objectives

- Row-based standard-cell placement
  - Cell heights are typically fixed, to fit in rows (but some cells may have double and quadruple heights)
  - Legal cell sites facilitate the alignment of routing tracks, connection to power and ground rails
- Wirelength as a key metric of interconnect
  - Bounding box half-perimeter (HPWL)
  - Cliques and stars
  - RMSTs and RSMTs
- Objectives: wirelength, routing congestion, circuit delay
  - Algorithm development is usually driven by wirelength
  - The basic framework is implemented, evaluated and made competitive on standard benchmarks
  - Additional objectives are added to an operational framework

# Summary of Chapter 4 – Global Placement

- Combinatorial optimization techniques: min-cut and simulated annealing
  - Can perform both global and detailed placement
  - Reasonably good at small to medium scales
  - SA is very slow, but can handle a greater variety of constraints
  - Randomized and chaotic algorithms – small changes at the input can lead to large changes at the output
- Analytic techniques: force-directed placement and non-convex optimization
  - Primarily used for global placement
  - Unrivaled for large netlists in speed and solution quality
  - Capture the placement problem by mathematical optimization
  - Use efficient numerical analysis algorithms
  - Ensure stability: small changes at the input can cause only small changes at the output
  - Example: a modern, competitive analytic global placer takes 20mins for global placement of a netlist with 2.1M cells (single thread, 3.2GHz Intel CPU) <sup>[1]</sup>

# Summary of Chapter 4 – Legalization and Detailed Placement

- Legalization ensures that design rules & constraints are satisfied
  - All cells are in rows
  - Cells align with routing tracks
  - Cells connect to power & ground rails
  - Additional constraints are often considered, e.g., maximum cell density
- Detailed placement reduces interconnect, while preserving legality
  - Swapping neighboring cells, rotating groups of three
  - Optimal branch-and-bound on small groups of cells
  - Sliding cells along their rows
  - Other local changes
- Extensions to optimize routed wirelength, routing congestion and circuit timing
- Relatively straightforward algorithms, but high-quality, fast implementation is important
- Most relevant after analytic global placement, but are also used after min-cut placement
- Rule of thumb: 50% runtime is spent in global placement, 50% in detailed placement [1]